

Advanced Structured Materials

G rard A. Maugin

Non-Classical Continuum Mechanics

A Dictionary

 Springer

Advanced Structured Materials

Volume 51

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A Dictionary

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ISSN 1869-8433 ISSN 1869-8441 (electronic)
Advanced Structured Materials
ISBN 978-981-10-2433-7 ISBN 978-981-10-2434-4 (eBook)
DOI 10.1007/978-981-10-2434-4

Library of Congress Control Number: 2016948287

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Printed on acid-free paper

This Springer imprint is published by Springer Nature

The registered company is Springer Nature Singapore Pte Ltd.

The registered company address is: 152 Beach Road, #22-06/08 Gateway East, Singapore 189721, Singapore

*“Young men should prove theorems. Old men
should write books”
(attributed to Godfrey Harold HARDY,
British mathematician, 1877–1947)*

Preface

This short preface has a main purpose to explain how I came to the idea of the present opus.

From my viewpoint, a short encyclopaedia or dictionary should give the basic definitions, main historic developments, a short technical description, directions of research, and a short selected but efficient bibliography for each item.¹ It should also point out unavoidable relationships between various entries, so that the redaction of such a work requires some technical experience from its author and also some benevolence and open mindedness. Because of my life story and experience I believe to have acquired the required conspectus, but also inquisitiveness, for this redaction in a selected subject matter, theories of continua that are decidedly *not classical*.

My experience in writing concise technical reports on various facets of science and technology goes back to a job I had to fulfil during my short stay in the French Air Force (since the French Ministry of Defence—note it's always “Defence”—had paid some of my studies). To say the truth I had to write reports on ongoing research in various countries from documents that were not always publicly accessible. This proved to be a good training. Now in my professional scientific career, which really started in 1968, I wrote an innumerable quantity of reports on already published papers (for *Mathematical Reviews*, *Applied Mechanics Reviews*, and *Zentralblatt für Mathematik*), more than seventy reviews (often as short essays) of published books, and also an incredible number of assessments of papers proposed for publication in many scientific journals relevant to continuum mechanics, applied mathematics and mathematical physics. I had the weakness practically

¹We are here faithful to d’Alembert’s preliminary discourse to the celebrated eighteenth-century grand encyclopaedia of Diderot and d’Alembert when this author wrote (cf. p. 4 in the English translation of “Preliminary discourse...” by R.N. Schwab, Bobbs-Merrill, Indianapolis, 1963): “As an Encyclopédie, it is set to forth as well as possible the order and connection of the parts of human knowledge. As reasoned Dictionary of the Sciences, Arts, and Trades, it is to contain the general principles that form the basis of each science and each art, liberal or mechanical, and the most essential facts that make up the body and substance of each”.

to never refuse this duty for two reasons. One is that this duty provides an easy way to keep aware of recent developments without having to investigate too much by oneself in the ocean of publications, the other being that someone must do the job, menial as it is, and it better be someone well informed, smart (here no false modesty) and benevolent than someone inexperienced and grumpy. Of course, this is not really creative work, but it is a way to remain a perpetual student. This is not also high style literature, but not everyone is a born Marcel Proust. Anyway, as they say: “Proust is too long and life is too short”. Smart editors-in-chief—from the UK, the USA, Germany, and France (I don’t give names)—succeeded in exploiting my somewhat naïve vanity by using arguments like: “Only you can look at this paper, only you can make some sense out of this mess, etc” that reminds me of a song of my youth “Only you—and you alone—can make the darkness bright” by the famous vocal group of singers called the *Fabulous Platters* in the 1950s and 1960s.

More specifically, concerning the very subject matter of this book, it happened that most of my creative scientific career—roughly the period 1970–2010—took place in a time that witnessed the burgeoning of new ideas and new models to describe the continuum mechanics of materials at different scales while my direct masters had contributed to the emergence of a new *generalized continuum mechanics (GCM)* in the 1960s and 1970s, and my own research took me to little explored (at the time) fields such as a true nonlinear continuum mechanics of electromagnetic solids, coupled linear and nonlinear waves in such fields, and so-called configurational mechanics with the accompanying paraphernalia of non-Riemannian geometry. I had the chance to witness some of these developments in *GCM* at Princeton and in Summer schools held at the time. There were busy years such as 1964 that saw the simultaneous publication of at least four different expansions in *GCM*—by Toupin, Mindlin, Eringen, and Green and Naghdi—with harsh confrontation between the different tenants in the late 1960s and early 1970s, and new approaches to continuously dislocated bodies. The 1970s were also rich with the development of nonlocal theories of various types. Other complex theories, such as those of porous bodies, superfluids, liquid crystals, extended thermodynamics, generalized thermo-elasticity, were also born. It is all these aspects in their diversity and also in what they share in common that is the true subject matter of this short book, with a will to help those confused readers and scientists new to the field to apprehend it in the best, albeit concise, conditions. To some of them it will open new horizons, to others it may correct some misinterpretations and favour a revisited fruitful interest. In all it should satisfy the natural scientific curiosity of many readers, who I expect to be perpetual students just like myself.

The work is presented in two parts. Part I includes prerequisites in classical continuum mechanics, and elements of the mechanics of generalized continua. This provides a necessary background and a general view of non-classical continuum mechanics, especially in the form of generalized continuum mechanics. Part II constitutes the dictionary *per se* in *alphabetic order* of the entries—so that there is no real need for a subject index. This includes around a hundred entries with numerous reference citations and cross references. These entries are of various sizes and in-depth description extending from a few lines to several pages. For the most

largely expanded ones, historical background is given (all original sources were consulted but transcribed in a modern unifying notation) as well as basic formulation, further progress, contemporary references, and cross references. I am sure that this is not exhaustive and any gross error and absence of relevant items are due only to my own focused idiosyncrasy and my negligence. I expect the reader to forgive me as the field is open and infinite by its very definition.

Paris, France
June 2016

Gérard A. Maugin

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Part I

Prerequisites