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Becoming a Mathematician

An international perspective

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Chapter 1
Introduction: How Does a Person Become a Mathematician?

What This Book Is About

People understand and experience the world in different ways. Some people believe that love, family and friendship are the most important aims in life, others would argue that that art, music and culture are most important, and yet others believe that science or football are critical for their existence. Mathematics pervades human experience and makes an impact in nearly all spheres of life. For some people, mathematics is only a passing inconvenience and they may pay very little attention to it at all. For others – and these are the people this book is about and for – mathematics, mathematical thinking and mathematical practices play a vital role in their personal and professional lives. This book is about how a person, often a student, goes through the process of becoming a mathematician, and how he or she comes to think of themselves as a mathematician. For some people, their identity is, or can be, aligned with the ideas and approaches of mathematics in an essential way; but this is not true of everyone. We explore this idea of mathematical identity from the perspective of students of mathematics, with an aim of learning from them and identifying the pedagogical approaches that foster a mathematical identity.

Over the course of more than a decade, with a number of colleagues in university education, we have grappled with various aspects of mathematics education. We have investigated our students' learning and our own pedagogical approaches, the curriculum we design and the mathematics that we present, the utility of mathematical knowledge and processes for learners' careers, and the concept of mathematical identity and what this implies for the process of becoming a mathematician. In the course of our investigations we have discovered (as many others have before us) that there are no simple solutions to the problems of how to help students to learn mathematics and to develop a mathematical identity. In this book we describe some of our approaches to these problems and some of the results we have obtained. And a key feature of these approaches and results is that they are firmly based on the experiences of mathematics learners, related to us in interviews and surveys in their own

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words, and included in most of our chapters in the form of verbatim quotes. In this way, our book also includes the voices of learners of mathematics and recent graduates from degrees in the mathematical sciences.

Why Is Mathematics Important?

A recent report on mathematics and statistics in Australia talks of *“the critical nature of mathematical sciences”*:

> The mathematical sciences are fundamental to the well-being of all nations. They drive the data analysis, forecasting, modelling, decision-making, management, design and technological principles that underpin nearly every sector of modern enterprise. Mathematics is the pre-eminent ‘enabling science’ that empowers research, development and innovation in business and industry, science and technology, national security and public health. Enabling sciences are, by their nature, often invisible to the wider community but without them, modern societies would cease to function. (Australian Academy of Science 2006, p. 18)

There is a fairly clear, if utilitarian, message here – and one that applies equally to other countries and will remain relevant well into the future. But we can compare this with an earlier description of the important aspects of mathematics, written by the mathematician G.H. Hardy a lifetime ago:

> A mathematician, like a painter or a poet, is a maker of patterns. If his [sic] patterns are more permanent that theirs, it is because they are made with *ideas*. A painter makes patterns with shapes and colours, a poet with words. … The mathematician’s patterns, like the painter’s or the poet’s must be beautiful; the ideas like the colours or the words, must fit together in a harmonious way. Beauty is the first test: there is no permanent place in the world for ugly mathematics. (Hardy 1940, section 10, pp. 13–14)

Our experience and use of mathematics in the early twenty-first century could be seen, in some respects, to be quite different from the situation in previous times. In the contemporary era of mass education, developing technologies, rapid communications and knowledge work, mathematics may be benefiting from a broader appreciation of its utility – and maybe also from its beauty, though it is likely that only a minority of people would have appreciated Hardy’s view even when it was written. A look at the declining numbers of school and university students choosing to study mathematics – in many other countries as well as Australia – suggests that for many people mathematics is not a glorious and fascinating subject. Nevertheless, most people will agree that mathematics lies at the heart of contemporary communication technologies, it plays an essential role in the world of finance, and it is indispensable in modern medicine. Exponents of mathematics would say that it lies at the very heart of oneself; and this is the point that causes educators the most concern. Despite the critical nature of mathematics, very few people consider mathematics to be central to their own identity. More than a decade of research with students has shown us that very few learners come to understand mathematics in this intimate way, and the same research shows that those who *don’t* often have a very hard time trying to work out how mathematics could be used for anything!

This brings us to a conundrum. Just why is mathematics important, to whom and for what? And how could we communicate this importance to learners of mathematics? A brief examination can illuminate some cultural attitudes towards mathematics. Firstly, populations usually only have access to the forms of knowledge about mathematics that are immediately applicable to their situations. For instance, early humans used physical objects to represent mathematical thinking and communication. A collection of rocks in a bowl could communicate how many sheep were in a field, or how many of them were to be exchanged for something else. Early literate societies could represent numbers with objects that were close to hand, such as stones, bones, ceramic pieces, notches in wood or knots in strings. The use of the phrase 'early literate' is deliberate in this sense and pertains to a form of mathematical literacy that is both conceptual and practical. The material of mathematics could include fingers or other body parts, attached to a single person or in groups of many persons, or the materials could be manufactured specifically to represent a concept of number.

Progressions in mathematical literacy are often generated by practical need, and each cultural group may respond in quite different ways to a similar need. In the Inca empire, financial and demographic information was recorded in the knotted strings called khipu, prepared by the expert 'khipukamayuqs' (see http://khipukamayuq.fas.harvard.edu) – maybe the original statisticians, and hence mathematical scientists. In fifteenth-century Europe, one focus of mathematics was bookkeeping, using the talents of some clever 'computers', people who were able to carry out arithmetical operations. The mathematician Luca Pacioli wrote a short treatise on the subject in 1494, introducing symbols for plus and minus. Our contemporary professions of accountant and actuary continue this line, keeping track of economic and financial transactions and the associated risks. The 'Songlines' of the Australian Aboriginals allowed them to navigate over large distances by referring to the words of the song, describing the geometry of the land by referring to locations of various landmarks. From the fifteenth century onwards, to support European voyages of exploration, mathematicians devoted much effort to the complex construction of tables which would help people to determine their location on the surface of a sphere; the problem was not completely solved until John Harrison completed the first true chronometer in 1735. Nowadays, GPS or Global Positioning System – a satellite-based navigation system – enables users to easily obtain their location anywhere on Earth, and Google Earth allows anyone with access to a computer to 'visit' any spot with equal ease.

As these examples indicate, mathematics seems to have a habit of infiltrating human activity, maybe in a form that is recognisably mathematical, but maybe just sitting behind the practical face of a pressing problem. In his communication of the architectural needs for the Colonia Güell chapel, the Spanish Catalan architect Antoni Gaudí utilised 'funicular models' (see Collins 1963, or the photos at http://www.gaudidesigner.com/uk/colonia-guell-construction.html) of dome stresses using string, shotgun pellets and gravity. Basically the dome of the chapel was hung upside down using string to 'draw' the catenary-shaped lines that were then followed by the masons. This simple idea was then used for the much-larger Sagrada Familia cathedral where the spaces were not regular. This architectural design approach was revolutionary and an example of implicit mathematical thinking (though the mathematical basis was very clear). Nevertheless, Gaudí's techniques were based on traditional Catalan tile-vaulting methods that stretched back to Roman times, or even Mesopotamian brick vaults, and they anticipated late-twentieth-century shell-vaulted methods of architecture. The results seem to combine the functional and aesthetic aspects of mathematics that are exemplified in the quotes earlier in this section.

It seems that people have always had a relationship with mathematics, and that the relationship is built on culture, interest and opportunity. One such opportunity arose in Europe in the nineteenth century when compulsory primary education resulted in a population that was literate. This led to a demand for different sorts of reading material: novels, newspapers, cartoons, letters, pamphlets and so on. In amongst these new texts, entertaining mathematics started to crop up. The logical puzzles of Lewis Carroll and Henry Ernest Dudeney, the mathematical games of Martin Gardiner in *Scientific American*, the statistical diversions of Petocz and Sowey (e.g., 2011) in *Teaching Statistics*, numerous mathematical competitions and Olympiads, and the contemporary love affair with Sudoku are all examples of this of this form of opportunity. By now we have come to an idea that mathematics can be in some way entertaining and enjoyable, as well as useful and beautiful.

In stark contrast to these examples of the importance of simple mathematics for everyday living, there have always been a few exceptional mathematicians who have been able to imagine, extend, speculate, experiment, deduce and create new forms of mathematics just for the sake of it – though it so often happens that applications are found even for such mathematics. G. H. Hardy thought he was on secure ground when he wrote: *"No one has yet discovered any warlike purpose to be served by the theory of numbers or relativity, and it seems very unlikely that anyone will do so for many years"* (Hardy 1940, section 28, p. 44). However, the development of cryptography after the second World War made extensive use of number theory, and contemporary public-key systems depend on Hardy's favourite branch of mathematics. All electronic financial interactions are now based on strong security measures that are purely mathematical, essentially the multiplication of extremely long primes. Most ordinary people simply accept that there must be some tricky maths going on somehow, but trust the system to work (in the same way that they would expect their car to work). Yet the designers of the systems are entranced by developing even more complex mathematical models to deal with security protocols and breaches. This in turn delights the mathematically inclined hackers who see it as a wonderful puzzle to solve that also destabilizes established hegemonies. This latter form could be seen as an instance of 'malevolent creativity', but perhaps it could also be seen as an emancipatory act.

Apple (1992) suggests that there are two forms of mathematical literacy, the functional and the critical. Functional mathematical literacy pertains to the applications of mathematics required in practical contexts, while critical mathematical literacy has the capacity to change the ways of thinking and even challenge society. The examples above show many instances of the functional, but also of the critical. In some senses, even the functional have the ability to change society because the job itself has a social effect. Take, for instance, the bookkeeper of eighteenth-century industrial Europe, a person who had an individual capacity for numeracy. However, today's accountancy profession embeds analysis of elements of financial risk, ethics, quality, financial growth and communication. Essentially, the bookkeeper's job has led to a change in society's understanding of risk and insurance. An individual can also develop a critical personal understanding of mathematics that challenges society. Compiling data provided by the United Nations, Hans Rosling created the online resource *Gapminder* (see http://www.gapminder.org). This investigative tool enables people to explore the world and 'mind the gap' between poverty and quality-of-life indicators for all nations. In this way, we can see the impact of a single mind on society, but also the intention to change society and individuals within it through the power of statistical graphics.

The potential of the mathematical sciences to raise awareness of social problems and maybe even change society for the better is another valid reason for the importance of mathematics. The Brazilian mathematics educator Ubiritan D'Ambrosio introduced the notion of 'ethnomathematics' to describe and legitimise the mathematics practiced among identifiable cultural groups that is often distinct from mainstream mathematics. As an exponent of 'critical mathematics education', he summarises his broad vision of the role of mathematics:

> No one will deny that the *most universal problem* is survival with dignity. Many people claim that mathematics is the *most universal mode of thought*. I believe that to find the relation between these two universals is an inescapable result of the claim of the universality of mathematics. (D'Ambrosio 1998, p. 68, original italics)

Much has been written on gender and the dominance of male views in mathematics (for example, the ICMI study edited by Hanna 2002), and also about the role of power and equity in mathematical endeavour (for example, the collection edited by Valero and Zevenbergen 2004). In some instances, mathematics can act as a form of liberation in which individuals or groups of people are able to transcend social expectations and excel in the discipline. This has been true of males for many years, and the past 50 years has allowed women to move into mathematics in increasing numbers. However, there are still barriers to becoming a mathematician due to gender and socioeconomic status and mathematics is poorer for it. Increasing the opportunities for talented mathematics students to become professional mathematicians will enhance mathematics itself.

This leaves us with several questions: Is it possible for people to move from one way of thinking about mathematics to another? Is such movement always in a positive direction? How can students move from a functional towards a critical approach to mathematical literacy? Is understanding of mathematics tied to potential job opportunities, or is it only possible through an interest in the mathematics itself? How can students of the discipline develop identity as mathematicians?

How People Come to Work with Mathematics

In the early twenty-first century, the majority of people in the 'developed' world now live in situations where technology is ubiquitous and mathematics forms a central component of a wide range of jobs and entertainments. It seems that most students entering formal higher education expect to be working in the area in which they studied (Reid et al. 2011). In Australia, where the authors of this book reside, the government's strategic goal for education is that by 2035 at least 40% of the population should have a tertiary degree, a recognition that higher education is needed for twenty-first-century life. There is an awareness that 'knowledge workers' are required in every field, and this knowledge work includes sophisticated understanding of mathematics for a whole range of professions. We firmly believe that the mathematical sciences are important to society, including industries, governments and individuals. Mathematics in all its guises of quantitative skills, analytic skills, logical thinking, quantitative problem solving and specific content, is a core capability for the majority of graduates, and high-level capability in mathematics is important for a smaller number of graduates.

The three international authors of the report *Mathematics and Statistics: Critical skills for Australia's future* explain the same point in this way:

> Mathematics and statistics permeate the complex fabric of developed societies, and mathematicians distinctively shape its texture. At once a domain of knowledge itself and the quantitative language for other fields, mathematics constantly evolves through research driven by the interplay of internal and external questions. As but one example, mathematicians give form and voice to ideas that help structure and deploy the flood of information transforming all of commerce, technology, medicine and daily life. The ever-expanding role of the mathematical sciences, and the strategic need for support of the associated research and training work, has been well recognised in our home countries [France, United States] and in other vibrant and developing economies. (Australian Academy of Science 2006, p. 1)

Comparable reviews of mathematics in the UK and Canada find that mathematics is critical to economic prosperity and that it seems to be under-utilised in industry (Hoyles et al. 2002; Fields Institute Annual Report 2005). Rubinstein (2006), in an article entitled *The crisis in maths in Australia*, also describes the importance of mathematical ideas to industry, adding that:

> The paradox is that although there is no 'mathematical industry' similar to the chemical industry or earth sciences in mining, it is also true that there is no non-mathematical industry. Every area requires or can benefit from mathematicians and statisticians to increase efficiency.

This illustrates a difficulty with the mathematical sciences. As 'enabling sciences', they can contribute to a wide range of industries and occupations. However, the form of the contribution is not as clear as, say, a mining engineer to a mining project. It is harder to form a mathematics community and more difficult for others to see what you do professionally as a mathematician.

The authors of the Australian report conclude that:

> Nurturing the health and securing the future of Australia's mathematical sciences hinges on: ensuring an adequate supply of properly qualified mathematics teachers for all levels of schooling; mathematics and statistics at universities being taught by qualified mathematicians and statisticians. (Australian Academy of Science 2006, p. 58)