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Traffic Networks as Information Systems

A Viability Approach

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We dedicate this book to our
dei ex machina,
Alexandre Bayen, Christian Claudel,
Gaël Désilles and Patrick Saint-Pierre
and to the next generation
around Nathalie and Nicolas Désilles
and Pierre-Cyril Aubin-Frankowski.

Preface

The purpose of this book is to study

1. Traffic networks as information systems advising velocities to vehicles at each time and position through celerity regulators we have to construct;
2. By using a viability approach.

Although the abstract nature of the mathematical approach of this book is common to several types of networks, synaptic networks, biological networks, economic networks, etc., this book focuses mainly on road networks to anchor the main concepts, enunciate some traffic problems and solve them. Road networks are indeed familiar to everyone, whereas other applications require some knowledge of cognitive, biological, and financial sciences, economics, etc.

For road systems, vehicles are driven by human *brains* that learned how to pilot the vehicles to link a departure position to an arrival one while staying on the road and/or undertaking other chores. Although no computer can do as well, at least, for the time, we may dream to replace them by automats, or, provide the drivers useful information for piloting their vehicles from one position to another.[1]

This information already exists such as traffic laws and regulations, traffic information, including speed limit signals, traffic lights, etc. It is provided by "institutions" such as road and traffic authorities or agencies, which we refer to as a *"traffic regulator."*[2] It designs general driving regulations, equips physical network with advices and/or even limits velocities used by the vehicles circulating on the network, imposing traffic lights, etc., enforcing them to control whether that they are respected, etc.

[1]To the best of our knowledge, this suggestion to regard mathematically a network as an information system has not been made yet.

[2]For example, air control for aerial traffic.

[Celerity Regulator] *Let us consider any mobile* **g***governed by its own second-order differential equation* $p''(t) = g(t, p(t), p'(t))$ *feeding on time, position, and velocity*. Therefore, to drive this vehicle, the pilot must know the time t, the position $p(t)$ ***and the velocity*** $p'(t)$.

The drivers have access to time and position at a first glance, *but have to learn or guess from experience the velocity* $p'(t)$. For that purpose, they must also have a view of the network, its topography, slopes and turns, congestion, fuel, etc., for having a safe evaluation of the velocity.

This missing information, ***velocity***, should be provided by the traffic regulator to indicate the velocities $p'(t) = r(t, p(t))$ advised to the vehicles which must be used by ***any*** vehicle **g** passing at time t and position $p(t) = p$.

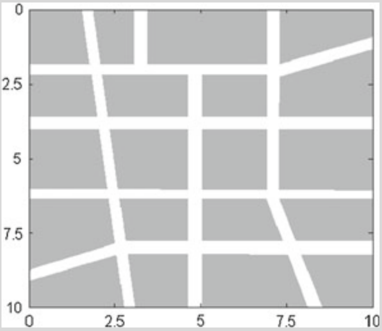
This *advised velocity* $r(t, p)$ is called the ***celerity***. It is provided by a *celerity regulator* $r : (t, p) \mapsto r(t, p)$ *computed* by the traffic regulator, *independently of individual vehicles*, but taking into account viability constraints and macroscopic traffic constraints.

Therefore, *abiding by the advised velocities* $r(t, p(t))$*provided by the celerity regulator, the only knowledge of time and position becomes sufficient* to satisfy the traffic requirements by coupling the *individual* dynamics of the vehicle **g** with the *collective* information furnished by the traffic regulator as follows: $p''(t) = g(t, p(t), r(t, p(t)))$.

In a near future, the recommended velocities provided by the celerity regulator could be posted on VMS (variable message signs), broadcast on mobile phones equipped with GPS, displayed on twinned speedometers measuring both the effective velocity and the recommended celerity (their difference triggering alarms), the latter being regulated by *cruise control systems* adjusting automatically the velocity to be equal to the broadcast celerity.[3]*Such information could be provided to the vehicles, which have to abide by this information for moving on the road.* Whether human beings comply with these advised celerities is outside the mathematical scope of this book, which does not address law and order issues.

For example, a network is described as a subset K of a vector space $\mathbb{R}^{|p|}$ on which vehicles circulate. An evolution $p(\cdot) : t \mapsto p(t)$ of the position of any vehicle is said to be *viable* if, at each instant t, its position $p(t) \in K$ stays on the network or, later, obeys other specific requirements.

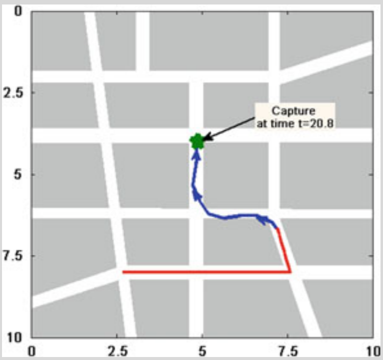
[3]For the time, cruise control systems adjust the velocity to a constant value entered by the driver provided by signed posted and valid for a road segment.



[Network] This example of a network $K \subset \mathbb{R}^2$ is a set of roads of a 2-dimensional network (in white). The first question studied in this book is to construct a *celerity regulator* indicating at each instant and at each position the velocity advised to any vehicle to remain on the road. We add other specifications (travel duration, acceleration, congestion, collision avoidance, etc.).

At this introductory stage, we mention only that the evolutions of positions are driven by *evolutionary systems* describing the set of viable evolutions of traffic positions which will be defined later.

Besides remaining in the road, the typical and central question is to find a viable evolution linking one departure position $s \in K$ to one arrival position $p \in K$ on the network, at a prescribed date T and for during a travel duration $\Omega \geq 0$. This means that we have to single out the subset $\mathcal{G}$ of quadruplets (T, Ω, s, p) such that an evolution arrives at date T at position p departing from position s at departure time $T - \Omega$.



[Geodesic Relation] Fixing the arrival date T, the travel duration Ω and the arrival position p, we wish to find a departure position s at departure time $T - \Omega$ from which starts an evolution viable on the road until it reaches p. The *geodesic relation* is the (possibly empty) subset $\mathcal{G}$ of (T, Ω, s, p) solving this problem. In this example, the vehicle starts from a departure position at departure time for reaching an arrival position signaled at arrival time.

The geodesic relation provides also other equivalent information, for instance:

1. If the arrival date T, the arrival position p and the departure position are prescribed, the travel durations $\Omega \geq 0$ are the ones such that $(T, \Omega, s, p) \in \mathcal{G}$;
2. If the arrival time T and the duration $\Omega \geq 0$ are fixed, the "geodesic pairs" (s, p) of departure and arrival positions are the ones such that $(T, \Omega, s, p) \in \mathcal{G}$,

as well as all allocations of these variables into two groups, the first one playing the role of inputs, the second one of outputs. Studying the relation between these four variables is the point of view we adopt: knowing a relation, its properties are shared by all the set-valued maps it generates.

We may and shall add later in the book other "traffic specifications" than staying on the road. For any traffic problem, the typical tasks assigned to us are to

1. Define the *relation* between the variables involved under which the evolutions satisfy the prescribed requirements;
2. Provide *intrinsic characterizations* of this relation and study their properties;
3. Construct (set-valued) *algorithms*[4] to compute them and to program their software;
4. And, above all, construct the *"celerity regulators,"* the very purpose of this study.

These are the main reasons why *the construction of celerity regulators, advising viable evolutions, satisfying the requirements, defining a relation* is the objective of all chapters of this book. The tools of viability theory allow us to design the celerity regulators advised to the vehicles. Although it is far to be familiar, *the "viability theorem" provides in a very simple*[5]*and economic way the celerity regulators* without using the intricacies of first-order partial differential equations or inclusions. Those can always be derived from the viability theorem, but we do not use them to derive the results we propose.

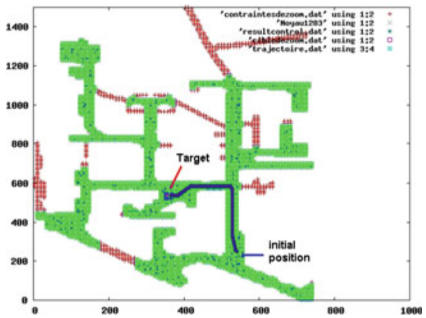
Paris Jean-Pierre Aubin
September 2015 Anya Désilles

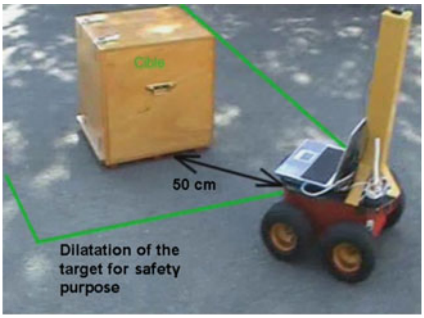
[4]Handling subsets, they belong to the field of "set-valued numerical analysis" which is building up.

[5]The price to pay is to become familiar with the basic results of set-valued analysis and to forget for a while differential calculus, before linking the two approaches. Unlearning is often more difficult than learning, since it requires a "waste" of acquired expertise and some dissidence, which is punished by the *law of perfect heresies* (see Box 173, Sect. 4.3, p. 271 of *La mort du devin, l'émergence du démiurge*, [20, Aubin]). Set-valued analysis goes back to *Pierre de Fermat* who invented tangents to sets, differential calculus to *Gottfried Leibniz* who introduced limits of differential quotients of functions, as we shall see. Set-valued analysis was first developed by the founding fathers of set theory before being banished in 1939 in favor of (single-valued) maps by *Bourbaki*.

Acknowledgments

The authors thank warmly *Guillaume Costesèque* for writing the panorama on macroscopic Hamiltonian models (Sect. 7.5, p.206) and bridging the active studies of road traffic to the viability approach as well as *Pierre-Cyril Aubin-Frankowski* for his proofreading of all the chapters of in this book. They are grateful also to *Luxi Chen*, *Olivier Dordan*, *Thomas Lorenz*, *John Lygeros*, *Sophie Martin*, *Marc Quincampoix*, and *Nicolas Seube* for their hidden contributions and thankful to *Alain Bensoussan*, *Philippe Boutry*[6], *Rémi Boutteau*, *Éva Crück*, *Marie-Hélène Durand*, *Nicolas Etienne*, *Hélène Frankowska*, *Alexandra Fronville*, *Benjamin Frenais de Coutard*, *Georges Haddad*, *Aurélien Gosselin*, *Stéphane Le Menec*, *Fabrice Leroux*, *Sylvain Rigal*, *Xavier Savatier*, *Nicolas Seube,* and *Pierre-Olivier Vandanjon* for their friendly disputations about the viability approach advocated in this book, without forgetting *Charles-Albert Lehalle* for the preliminary discussions on economic and financial networks.





[6]Head of VIMADES (Viabilité, Marchés, Automatique et Décision).

Reaching a Target While Avoiding Obstacles
This adventure started in 2003 when *Alexandre Bayen* approached the "Viabilité-Jeux-Contrôle" research group with the LWR and Moskowitz equations as a gift to play with. The first experiment was developed in 2005, thanks to the contract REI 04C0132 at the Laboratoire de Recherches Balistiques et Aérodynamiques (LRBA) headed by *Gaël Désilles*, with the collaboration of *Alexandre Bayen*, *Éva Crück*, *Benjamin Frenais de Coutard*, *Aurélien Gosselin*, *Fabrice Leroux*, *Sylvain Rigal*, *Patrick Saint-Pierre*, and other engineers.

The left figure displays the trajectory of the robot in the map of an urban network, the input for the software. The right figure is a close-up photograph of the "pioneer" near the target.

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