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Supersymmetric Gravity and Black Holes

Proceedings of the INFN-Laboratori Nazionali di Frascati School on the Attractor Mechanism 2009

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Mechanism 2009

With 16 Figures

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To Mario, a great physicist, a model of loyalty, integrity, coherence, and just the best Director I ever met.

Preface

This book is based upon lectures held from 29 June to 3 July 2009 at the INFN-Laboratori Nazionali di Frascati School on Attractor Mechanism, directed by Stefano Bellucci, with the participation of prestigious lecturers, including M. Cvetic, G. DallAgata, S. Ferrara, J.F. Morales, G. Moore, A. Sen, J. Simon, and M. Trigiante, as well as invited scientists of the caliber of M. Bianchi, C. Nappi, A. Sagnotti, and E. Witten. All lectures were given at a pedagogical, introductory level, a feature which reflects itself in the specific "flavor" of this volume, which also benefited much from extensive discussions and related reworking of the various contributions.

This is the fifth volume in a series of books on the general topics of supersymmetry, supergravity, black holes, and the attractor mechanism. Indeed, based on previous meetings, four volumes were already published:

Bellucci S. (2006). Supersymmetric Mechanics – Vol. 1: Supersymmetry, Noncommutativity and Matrix Models. (vol. 698, pp. 1–229). ISBN: 3-540-33313-4. (Springer, Berlin Heidelberg) Lecture Notes in Physics Vol. 698.

Bellucci S., S. Ferrara, A. Marrani. (2006). Supersymmetric Mechanics – Vol. 2: The Attractor Mechanism and Space Time Singularities. (vol. 701, pp. 1–242). ISBN: 978-3-540-34156-7. (Springer, Berlin Heidelberg) Lecture Notes in Physics Vol. 701.

Bellucci S. (2008). Supersymmetric Mechanics – Vol. 3: Attractors and Black Holes in Supersymmetric Gravity. (vol. 755, pp. 1–373). ISBN: 978-3-540-79522-3. (Springer, Berlin Heidelberg) Lecture Notes in Physics Vol. 755.

Bellucci S. (2010). The Attractor Mechanism. Proceedings of the INFN-Laboratori Nazionali di Frascati School 2007. ISSN 0930-8989, ISBN 978-3-642-10735-1, e-ISBN 978-3-642-10736-8. DOI 10.1007/978-3-642-10736-8. (Springer Heidelberg Dordrecht London New York) Proceedings in Physics Vol. 134.

I wish to thank all lecturers, invited scientists, and participants at the School for contributing to the success of the School, which prompted the realization of this volume. I wish to thank especially Mario Calvetti for giving vital support to the School and for personal trust and enduring encouragement. Lastly, but most importantly, my gratitude goes to my wife Gloria and our beloved daughters Costanza, Eleonora, Annalisa, Erica, and Maristella for love and inspiration, in want of which I would have never had the strength to complete this book.

Frascati, Italy Stefano Bellucci

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Chapter 1
Black Holes in Supergravity: Flow Equations and Duality

Gianguido Dall'Agata

1.1 Introduction

The analysis of black hole solutions and the study of their physics is an active and important branch of contemporary theoretical physics. In fact, not only black holes are an excellent theoretical laboratory for understanding some features of quantum gravity, but they can also be successfully used as a tool in applications to nuclear physics, condensed matter, algebraic geometry and atomic physics. For this reason, black holes are considered the "Hydrogen atom" of quantum gravity [67] or the "harmonic oscillator of the 21st century" [77].

The existence of black holes seems to be an unavoidable consequence of General Relativity (GR) and of its extensions (like supergravity). Classically, the horizon of black holes protects the physics in the outer region from what happens in the vicinity of singular field configurations that can arise in GR from smooth initial data. However, already at the semiclassical level, black holes emit particles with a thermal spectrum [7, 58]. A thermodynamic behaviour can also be associated to black holes from the laws governing their mechanics [79] and, in particular, one can associate to a black hole an entropy S proportional to the area A of its event horizon (measured in Planck units $l_P^2 = G\hbar/c^3$)

$$S = \frac{k_B}{l_P^2}\,\frac{A}{4}. \tag{1.1}$$

In most physical systems the thermodynamic entropy has a statistical interpretation in terms of counting microscopic configurations with the same macroscopic

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properties, and in most cases this counting requires an understanding of the quantum degrees of freedom of the system. The identification of the degrees of freedom that the Bekenstein–Hawking entropy is counting is a long-standing puzzle that motivated much theoretical work of the last few years. String Theory, being a theory of quantum gravity, should be able to provide a microscopic description of black holes and hence justify Bekenstein–Hawking's formula. By now we have strong indications and many different and compelling examples where String Theory successfully accomplishes this goal, although often simplifying assumptions are made so that the configurations which are considered are not very realistic. In particular, black holes are non-perturbative objects and only for special classes of solutions (mainly supersymmetric) string theory at weak coupling can reproduce the correct answer[1] [33, 73, 78]. However, there is now a growing evidence that also for non-zero coupling we can identify candidate microstate geometries, whose quantization may eventually yield an entropy that has the same parametric dependence on the charges as that of supersymmetric black holes [5, 13, 65, 68].

In the last few years a lot of progress has been made in understanding the physics of *extremal non-supersymmetric solutions* and of their candidate microstates. The aim of these lectures is to provide an elementary and self-contained introduction to supergravity black holes, describing in detail the techniques that allow to construct full extremal solutions and to discuss their physical properties. We will especially focus on the peculiar role of scalar fields in supergravity models and on the flow equations driving them to the attractor point provided by the black hole horizon. We will also discuss the multicentre solutions and the role of duality transformations in establishing the classes of independent solutions.

1.2 Black Holes and Extremality

In this section we will review some general properties of black holes and discuss the concept of *extremality*, both in the context of geometrical and of thermodynamical properties of the solutions.

We will be interested in charged black hole configurations, so our starting point is the Einstein–Maxwell action in 4 dimensions, with Lagrangian density given by

$$e^{-1}\mathcal{L} = R - \frac{1}{4}\, F_{\mu\nu} F^{\mu\nu}. \tag{1.2}$$

For the sake of simplicity we will look for static, spherically symmetric and charged solutions. This means that the line element describing the metric should be of the form

[1] Recently there has been also a lot of progress in understanding the nature of the entropy for Kerr black holes and close to extremal examples of this sort can be realized in nature [20].

$$ds^2 = -e^{2U(r)}dt^2 + e^{-2U(r)}dr^2 + r^2 d\Omega^2, \tag{1.3}$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta\, d\phi^2$ is the line element of a two-sphere and U is the warp factor, which depends only on the radial variable in order to respect spherical symmetry. For the same reason, the two-form associated to the Maxwell field $F_{\mu\nu}$ should be of the form

$$F = P\ \sin\theta\, d\theta \wedge d\phi + Q\, dt \wedge \frac{dr}{r^2}, \tag{1.4}$$

so that, by integrating over a sphere, one gets the electric and magnetic charge of the configuration:

$$\frac{1}{4\pi}\int_{S^2} F = P\,, \qquad \frac{1}{4\pi}\int_{S^2} \star F = Q. \tag{1.5}$$

By solving the equations of motion derived from (1.2) we obtain the following expression for the warp factor

$$e^{2U(r)} = 1 - \frac{2M}{r} + \frac{P^2 + Q^2}{r^2}, \tag{1.6}$$

which is the appropriate one for a Reissner–Nordström black hole and reduces to the one by Schwarzschild for $P = Q = 0$.

The solution above contains a singularity at $r = 0$, as one can see by computing the quadratic scalar constructed in terms of the Ricci tensor

$$R_{\mu\nu}R^{\mu\nu} = 4\frac{(Q^2 + P^2)^2}{r^8} \overset{r\to 0}{\longrightarrow} \infty \tag{1.7}$$

(For the special case $P = Q = 0$ we can still find a singularity in $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} = 48\frac{M^2}{r^6}$). However, the singularity is hidden by the horizons appearing at the zeros of the warp-factor function

$$e^{2U} = 0 \quad \Leftrightarrow \quad r_{\pm} = M \pm \sqrt{M^2 - (P^2 + Q^2)}. \tag{1.8}$$

The two solutions are real as long as $M^2 \geq P^2 + Q^2$, while the singularity becomes naked for smaller values of the mass. This means that, *for fixed charges, there is a minimum value of the mass for which the singularity is screened by the horizons*. At such value the warp factor has a double zero, the two horizons coincide and the semi-positive definite parameter

$$c = r_+ - r_- = \sqrt{M^2 - (P^2 + Q^2)}\,, \tag{1.9}$$

which we introduce for convenience, is vanishing. The corresponding black hole configuration is called *extremal* ($c = 0$ or $M = \sqrt{P^2 + Q^2}$). Note that in the

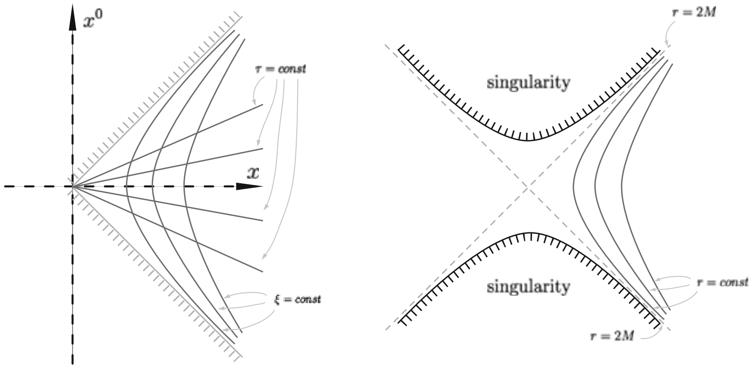

Fig. 1.1 Minkowski and Schwarzschild spacetimes in Rindler coordinates. The first diagram approximates the second close to the horizon

uncharged limit $c = M$, which is the extremality parameter for the Schwarzschild solution. This means that extremal Schwarzschild black holes are necessarily *small*, i.e. with vanishing horizon area at tree level.

Although the singularity is timelike (for charged solutions) and hence one can interpret it as the presence of a source, the existence of the horizons guarantees that the physics outside the horizon is not influenced by what happens inside, where one meets the singularity. This fact is easily seen by computing the time it takes for a light ray traveling radially to reach the horizon from infinity, as measured by an observer sitting far from the black hole. By taking $ds = 0$ for constant θ and ϕ one gets that

$$\sqrt{g_{tt}}\, dt = \sqrt{g_{rr}}\, dr, \tag{1.10}$$

so that the time it takes for a light ray to travel radially between two points at distance r_1 and r_2 from the singularity is proportional to the distance measured with a weight given by the inverse of the warp factor

$$t_{12} = \int_{r_1}^{r_2} \sqrt{\frac{g_{rr}}{g_{tt}}} d\tilde{r} = \int_{r_1}^{r_2} e^{-2U(\tilde{r})} d\tilde{r}. \tag{1.11}$$

This expression goes to infinity when $r_1 \to r_+$ and therefore a signal from the horizon takes an infinite time to reach a far distant observer.

The physics close to the horizon can be better understood by considering the expansion of the solution obtained above for r close to r_+ (Fig. 1.1). The only non-trivial function in the metric is given by the warp factor, which approaches

$$e^{2U} = \frac{(r - r_+)(r - r_-)}{r^2} \quad \stackrel{r \to r_+}{\longrightarrow} \quad \frac{r_+ - r_-}{r_+^2}\, \rho, \tag{1.12}$$

where we introduced a new coordinate ρ measuring the distance from the outer horizon: $\rho = r - r_+$. The resulting near horizon geometry is

$$ds^2 \to -\frac{r_+ - r_-}{r_+^2}\rho dt^2 + \frac{r_+^2}{r_+ - r_-}\frac{d\rho^2}{\rho} + r_+^2\, d\Omega^2, \tag{1.13}$$

which can be interpreted as the product of a 2-dimensional Rindler spacetime with a two-sphere of radius r_+. We can actually make this result explicit by performing another change of coordinates $(t, \rho) \mapsto (\tau, \xi)$ as follows

$$\rho = e^{2\alpha\xi}, \qquad t = \frac{1}{4\alpha^2}\tau, \qquad \alpha = \frac{\sqrt{r_+ - r_-}}{2r_+}. \tag{1.14}$$

This leads to a near-horizon metric described by

$$ds^2_{NH} = e^{2\alpha\xi}\left(-d\tau^2 + d\xi^2\right) + r_+^2\, d\Omega^2. \tag{1.15}$$

The geometry of the non-compact part is 2-dimensional Minkowski spacetime as seen by an observer that is uniformly accelerated with acceleration $\alpha = \sqrt{\alpha_\mu \alpha^\mu}$. In fact the change of coordinates from the standard ones to Rindler's is dictated by the trajectory of an accelerated observer

$$x(x^0) = \frac{1}{\alpha}\sqrt{1 + \alpha^2 (x^0)^2}, \tag{1.16}$$

and τ denotes the proper time

$$x^0(\tau) = \frac{1}{\alpha}\sinh(\alpha\tau). \tag{1.17}$$

Our derivation explains this acceleration as the effect of gravitation and one can actually show that α coincides with the surface gravity of the black hole. In fact surface gravity is given in terms of the derivative of the null Killing vector generating the horizon surface, computed at the surface [79]

$$\alpha^2 = \left[-\frac{1}{2}\nabla_\mu \xi_\nu \nabla^\mu \xi^\nu\right]_{r=r_+} \tag{1.18}$$

and the two expressions coincide.

1.2.1 Thermodynamics

Hawking and Unruh showed that an accelerated observer following the trajectory described in (1.16) sees a thermal spectrum with temperature proportional to the acceleration:

$$T = \frac{\alpha}{2\pi}. \tag{1.19}$$