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Linear Estimation and Detection in Krylov Subspaces

Guido K.E. Dietl *Editor*

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To Anja

Preface

One major area in the theory of statistical signal processing is reduced-rank estimation where optimal linear estimators are approximated in low-dimensional subspaces, e. g., in order to reduce the noise in overmodeled problems, enhance the performance in case of estimated statistics, and/or save computational complexity in the design of the estimator which requires the solution of linear equation systems. This book provides a comprehensive overview over reduced-rank filters where the main emphasis is put on matrix-valued filters whose design requires the solution of linear systems with multiple right-hand sides. In particular, the multistage matrix Wiener filter, i. e., a reduced-rank Wiener filter based on the multistage decomposition, is derived in its most general form.

In numerical mathematics, iterative block Krylov methods are very popular techniques for solving systems of linear equations with multiple right-hand sides, especially if the systems are large and sparse. Besides presenting a detailed overview of the most important block Krylov methods in Chapter 3, which may also serve as an introduction to the topic, their connection to the multistage matrix Wiener filter is revealed in this book. Especially, the reader will learn the restrictions of the multistage matrix Wiener filter which are necessary in order to end up in a block Krylov method. This relationship is of great theoretical importance because it connects two different fields of mathematics, viz., statistical signal processing and numerical linear algebra.

This book mainly addresses readers who are interested in the theory of reduced-rank signal processing and block Krylov methods. However, it includes also practical issues like efficient algorithms for direct implementation or the exact computational complexity in terms of the required number of floating point operations. If the reader is not interested in these practical aspects, Sections 2.2, 4.3, and 4.4 of this book can be skipped.

Finally, the book covers additionally the application of the proposed linear estimators to a detection problem occurring at the receiver of a digital communication system. An iterative (Turbo) multiuser detector is considered where users are separated via spread spectrum techniques. Besides using

Monte Carlo simulations, the communication system is investigated in terms of the expected iterative estimation error based on extrinsic information transfer charts. It should be mentioned that the extrinsic information transfer characteristics that are shown in these charts, are calculated in a semianalytical way as derived in Section 6.1.2.

This text has been written at the Associate Institute for Signal Processing, Munich University of Technology, Germany, where I was working as a research engineer towards my doctoral degree. I would like to express my deep gratitude to Prof. Wolfgang Utschick who supervised me at the institute. I appreciate his helpful advice and steady support, as well as the numerous discussions which had a great impact on this book. Besides, I thank Prof. Michael L. Honig of the Northwestern University, USA, and Prof. Joachim Hagenauer of the Institute for Communications Engineering, Munich University of Technology, both members of my dissertation committee, for reviewing this manuscript. I also thank Prof. Michael D. Zoltowski of Purdue University, USA, for giving me the opportunity to stay at Purdue University in winter 2000/2001 and summer 2004. In fact, he initiated my research on the multistage Wiener filter and Krylov methods. Further, I would like to thank Prof. Josef A. Nossek of the Institute for Circuit Theory and Signal Processing, Munich University of Technology, for his support.

Finally, many thanks to all the excellent students which I had the chance to supervise at the Munich University of Technology. Their research results deeply influenced this book. Moreover, I thank all my colleagues at the institute for the nice atmosphere and the inspiring discussions.

Munich, March 2007 *Guido Dietl*

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