

Thorsten M. Buzug

# Computed Tomography

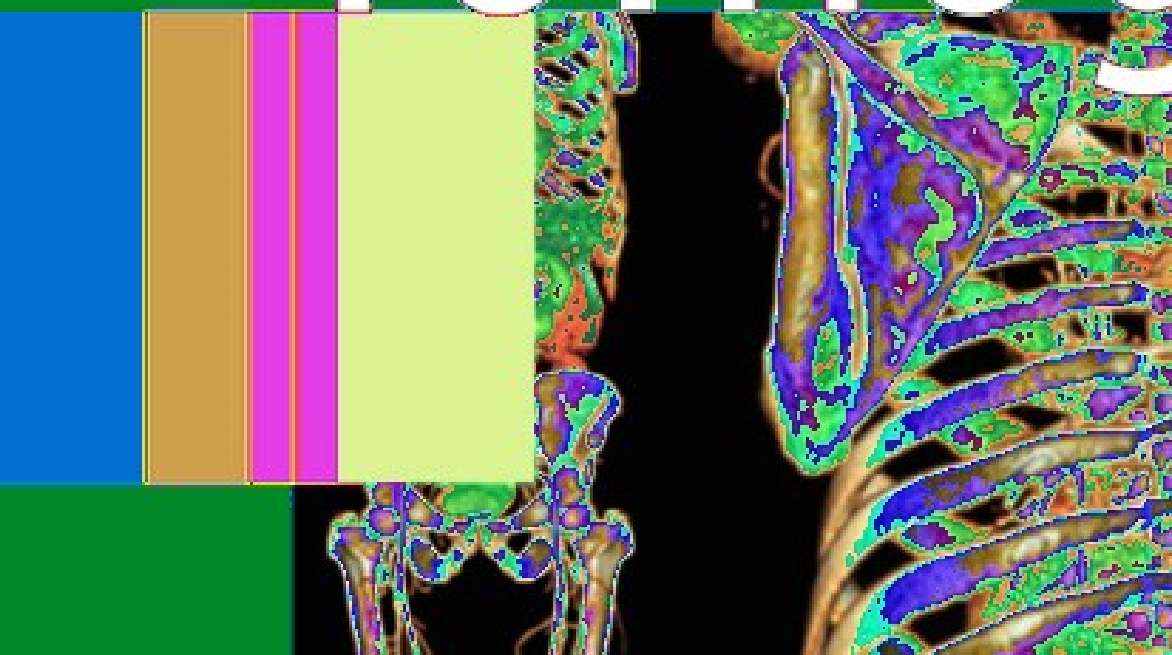


From Photon Statistics to  
Modern Cone-Beam CT

 Springer

Thorsten M. E.

# Compu Tomoc



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From Photon Statistics  
to Modern Cone-Beam CT

With 475 Figures and 10 Tables

 Springer

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# Preface

This book provides an overview of X-ray technology, the historic developmental milestones of modern CT systems, and gives a comprehensive insight into the main reconstruction methods used in computed tomography. The basis of reconstruction is, undoubtedly, mathematics. However, the beauty of computed tomography cannot be understood without a detailed knowledge of X-ray generation, photon-matter interaction, X-ray detection, photon statistics, as well as fundamental signal processing concepts and dedicated measurement systems. Therefore, the reader will find a number of references to these basic disciplines together with a brief introduction to the underlying principles of CT.

This book is structured to cover the basics of CT: from photon statistics to modern cone-beam systems. However, the main focus of the book is concerned with detailed derivations of reconstruction algorithms in 2D and modern 3D cone-beam systems. A thorough analysis of CT artifacts and a discussion of practical issues, such as dose considerations, provide further insight into modern CT systems. While mainly written for graduate students in biomedical engineering, medical engineering science, medical physics, medicine (radiology), mathematics, electrical engineering, and physics, experienced practitioners in these fields will benefit from this book as well.

The didactic approach is based on consistent notation. For example, the notation of computed tomography is used in the signal processing chapter. Therefore, contrary to many other signal processing books, which use time-dependent values, this book uses spatial variables in one, two or three dimensions. This facilitates the application of the mathematics and physics learned from the earlier chapters to detector array signal processing, which is described in the later chapters. Additionally, special attention has been paid to creating a text with detailed and richly discussed algorithm derivations rather than compact mathematical presentations. The concepts should give even undergraduate students the chance to understand the principal reconstruction theory of computed tomography. The text is supported by a large number of illustrations representing the geometry of the projection situation. Since the impact of cone-beam CT will undeniably increase in the future, three-dimensional reconstruction algorithms are illustrated and derived in detail.

This book attempts to close a gap. There are several excellent books on medical imaging technology that give a comprehensive overview of modern X-ray technology, computed tomography, magnetic resonance imaging, ultrasound, or nuclear

medicine modalities like PET and SPECT. However, these books often do not go into the mathematical detail of signal processing theory. On the other hand, there are a number of in-depth mathematical books on computed tomography that do not discuss practical issues. The present book is based on the German book *Einführung in die Computertomographie*, which first appeared during the summer of 2004. Fortunately, since the book was used by many of my students in lectures on *Engineering in Radiology, Medical Engineering, Signals and Systems in Medicine*, and *Tomographic Methods*, I received a lot of feedback regarding improvements on the first edition. Therefore, the idea arose to publish an English version of the book, which is a corrected and extended follow-up.

I would like to thank Siemens Medical Solutions, General Electric Medical Systems, and Philips Medical Systems, who generously supported my laboratories in the field of computed tomography. In particular, I would like to thank my friend Dr. Michael Kuhn, former Director of Philips Research Hamburg. It was his initiative that made possible the first installation of CT in my labs in 1999. Additionally, I have to thank Mrs. Annette Halstrick and Dr. Hans-Dieter Nagel (Philips Medical Systems Hamburg), Leon de Vries (Philips Medical Systems Best), Doris Pischitz, Jürgen Greim and Robby Rokkitta (Siemens Medical Solutions Erlangen), Dieter Keidel and Jan Liedtke (General Electric Medical Systems) for many photos in this book. I would like to thank Wolfgang Härer (AXI CC, Siemens Medical Solutions), Dr. Gerhard Brunst, (General Electric Medical Systems), Dr. Armin H. Pfoh, Director of General Electric Research Munich, Dr. Wolfgang Niederlag (Hospital Dresden-Friedrichstadt), Prof. Dr. Heinz U. Lemke (Technical University Berlin), Dr. Henrik Turbell (Institute of Technology, Linköpings Universitet), and Prof. Dr. Dr. Jürgen Ruhlmann (Medical Center Bonn) for the courtesy to allow me to use their illustrations and photos. Further, I have to thank the Digital Collections and Archives of Tufts University, the Collection of Portraits of the Austrian Academy of Sciences, and the Röntgen-Kuratorium Würzburg e.V. for the courtesy to allow me to use their photos.

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Lübeck, June 2008

Thorsten M. Buzug



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# 1 Introduction

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## 1.1 Computed Tomography – State of the Art

Computed tomography (CT) has evolved into an indispensable imaging method in clinical routine. It was the first method to non-invasively acquire images of the inside of the human body that were not biased by superposition of distinct anatomical structures. This is due to the projection of all the information into a two-dimensional imaging plane, as typically seen in planar X-ray fluoroscopy. Therefore, CT yields images of much higher contrast compared with conventional radiography. During the 1970s, this was an enormous step toward the advance of diagnostic possibilities in medicine.

However, research in the field of CT is still as exciting as at the beginning of its development during the 1960s and 1970s; however, several competing methods exist, the most important being magnetic resonance imaging (MRI). Since the invention of MRI during the 1980s, the phasing out of CT has been anticipated. Nevertheless, to date, the most widely used imaging technology in radiology departments is still CT. Although MRI and positron emission tomography (PET) have been widely installed in radiology and in nuclear medicine departments, the term tomography is clearly associated with X-ray computed tomography<sup>1</sup>.

Some hospitals actually replace their conventional shock rooms with a CT-based virtual shock room. In this scenario, imaging and primary care of the patient takes place using a CT scanner equipped with anesthesia devices. In a situation

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<sup>1</sup> In the United States computed tomography is also called CAT (computerized axial tomography).

where the fast three-dimensional imaging of a trauma patient is necessary (and it is unclear whether MRI is an adequate imaging method in terms of compatibility with this patient), computed tomography is the standard imaging modality. Additionally, due to its ease of use, clear interpretation in terms of physical attenuation values, progress in detector technology, reconstruction mathematics, and reduction of radiation exposure, computed tomography will maintain and expand its established position in the field of radiology.

Furthermore, the preoperatively acquired CT image stack can be used to synthetically compute projections for any given angulations. A surgeon can use this information in order to get an impression of the images that are taken intraoperatively by a C-arm image intensifier. Therefore, there is no need to acquire additional radiographs and the artificially generated projection images actually resemble conventional radiographs. Additionally, the German Employer's Liability Insurance Association insists on a CT examination in severe accidents that occur at work. Therefore, CT has advanced to become the standard diagnostic imaging modality in trauma clinics. Patients with heavy trauma, fractures, and luxations benefit greatly from the clarification provided by imaging techniques such as computed tomography.

Recently, interesting technical, anthropomorphic, forensic, and archeological (Thomsen et al. 2003) as well as paleontological (Pol et al. 2002) applications of computed tomography have been developed. These applications further strengthen the method as a generic diagnostic tool for non-destructive material testing and three-dimensional visualization beyond its medical use. Magnetic resonance imaging fails whenever the object to be examined is dehydrated. In these circumstances, computed tomography is the three-dimensional imaging method of choice.

## 1.2 Inverse Problems

The mathematics of CT image reconstruction has influenced other scientific fields and vice versa. The backprojection technique, for instance, is used in both geophysics and radar applications (Nilsson 1997). Clearly, the fundamental problem of computed tomography can be easily described: Reconstruct an object from its shadows or, more precisely, from its projections. An X-ray source with a fan- or cone-beam geometry penetrates the object to be examined as a patient in medical applications, a skull found in archeology or a specimen in nondestructive testing (NDT). In the so-called third generation scanners, the fan-shaped X-ray beam fully covers a slice section of the object to be examined.

Depending on the particular paths, the X-rays are attenuated at varying extents when running through the object; the local absorption is measured with a detector array. Of course, the shadow that is cast in only one direction is not an adequate basis for the determination of the spatial distribution of distinct structures inside a three-dimensional object. In order to determine this structure, it is necessary to