

International Perspectives on the Teaching and  
Learning of Mathematical Modelling

Gloria Ann Stillman

Werner Blum

Maria Salett Biembengut *Editors*

# Mathematical Modelling in Education Research and Practice

Cultural, Social and Cognitive Influences



Springer

# **International Perspectives on the Teaching and Learning of Mathematical Modelling**

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ICTMA is a worldwide unique group, in which not only mathematics educators aiming for education at school level are included but also applied mathematicians interested in teaching and learning modelling at tertiary level are represented. ICTMA discusses all aspects related to Teaching and Learning of Mathematical Modelling at Secondary and Tertiary Level, e.g. usage of technology in modelling, psychological aspects of modelling and its teaching, modelling competencies, modelling examples and courses, teacher education and teacher education courses.

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Gloria Ann Stillman • Werner Blum  
Maria Salett Biembengut  
Editors

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## Series Preface

Applications and modelling and their learning and teaching in schools and universities have become a prominent topic in the last decades in view of the growing worldwide relevance of the usage of mathematics in science, technology and everyday life. However, although there is consensus that modelling should play an important role in mathematics education, the situation in schools and universities is less than ideal in many educational jurisdictions. Given the worldwide impending shortage of students who are interested in mathematics and science, it is essential to discuss possible changes of mathematics education in school and tertiary education towards the inclusion of real-world examples and the competencies to use mathematics to solve real-world problems.

This innovative book series *International Perspectives on the Teaching and Learning of Mathematical Modelling* established by Springer aims at promoting academic discussion on the teaching and learning of mathematical modelling at various educational levels all over the world. The series will publish books from different theoretical perspectives from around the world dealing with Teaching and Learning of Mathematical Modelling in Schooling and at tertiary level. This series will also enable the *International Community of Teachers of Mathematical Modelling and Applications* (ICTMA), an International Commission on Mathematical Instruction-affiliated study group, to publish books arising from its biennial conference series. ICTMA is a unique worldwide group where not only mathematics educators dealing with education at school level are included but also applied mathematicians interested in teaching and learning modelling at tertiary level are represented. Three of these books published by Springer have already appeared.

The planned books will display the worldwide state of the art in this field, most recent educational research results and new theoretical developments and will be of interest for a wide audience. Themes dealt with in the books will be teaching and learning of mathematical modelling in schooling and at tertiary level including the usage of technology in modelling; psychological, social and cultural aspects of modelling and its teaching; modelling competencies; curricular aspects; modelling

examples and courses; teacher education; and teacher education courses. The book series aims to support the discussion on mathematical modelling and its teaching internationally and will promote the teaching and learning of mathematical modelling and research of this field all over the world in schools and universities.

The series is supported by an editorial board of internationally well-known scholars, who bring their long experience in the field as well as their expertise to this series. The members of the editorial board are Maria Salett Biembengut (Brazil), Werner Blum (Germany), Helen Doerr (USA), Peter Galbraith (Australia), Toshikazu Ikeda (Japan), Mogens Niss (Denmark) and Jinxing Xie (China).

We hope this book series will inspire readers in the present and the future to promote the teaching and learning of mathematical modelling all over the world.

Ballarat, Australia  
Hamburg, Germany

Gloria Ann Stillman  
Gabriele Kaiser

# Contents

<b>1</b>	<b>Cultural, Social, Cognitive and Research Influences on Mathematical Modelling Education . . . . .</b>	<b>1</b>
	Gloria Ann Stillman, Werner Blum, and Maria Salett Biembengut	
<b>Part I Innovative Practices in Modelling Education Research and Teaching</b>		
<b>2</b>	<b>Mathematical Modelling as a Strategy for Building-Up Systems of Knowledge in Different Cultural Environments . . . . .</b>	<b>35</b>
	Ubiratan D'Ambrosio	
<b>3</b>	<b>The Meaning of the Problem in a Mathematical Modelling Activity . . . . .</b>	<b>45</b>
	Lourdes Maria Werle de Almeida and Karina Alessandra Pessoa da Silva	
<b>4</b>	<b>Extending the Reach of the Models and Modelling Perspective: A Course-Sized Research Site . . . . .</b>	<b>55</b>
	Corey Brady, Richard Lesh, and Serife Sevis	
<b>5</b>	<b>Prescriptive Modelling – Challenges and Opportunities . . . . .</b>	<b>67</b>
	Mogens Niss	
<b>6</b>	<b>An Approach to Theory Based Modelling Tasks . . . . .</b>	<b>81</b>
	Xenia-Rosemarie Reit and Matthias Ludwig	
<b>7</b>	<b>Facilitating Mathematisation in Modelling by Beginning Modellers in Secondary School . . . . .</b>	<b>93</b>
	Gloria Ann Stillman, Jill P. Brown, and Vince Geiger	
<b>8</b>	<b>Authenticity in Extra-curricular Mathematics Activities: Researching Authenticity as a Social Construct . . . . .</b>	<b>105</b>
	Pauline Vos	



<b>9</b>	<b>The Teaching Goal and Oriented Learning of Mathematical Modelling Courses</b> . . . . .	115
	Mengda Wu, Dan Wang, and Xiaojun Duan	
<b>Part II Research into, or Evaluation of, Teaching and Learning</b>		
<b>10</b>	<b>Modelling Competencies: Past Development and Further Perspectives</b> . . . . .	129
	Gabriele Kaiser and Susanne Brand	
<b>11</b>	<b>How to Support Teachers to Give Feedback to Modelling Tasks Effectively? Results from a Teacher-Training-Study in the Co<sup>2</sup>CA Project</b> . . . . .	151
	Michael Besser, Werner Blum, and Dominik Leiss	
<b>12</b>	<b>A Reflection on Mathematical Modelling and Applications as a Field of Research: Theoretical Orientation and Diversity</b> . . . . .	161
	Vince Geiger and Peter Frejd	
<b>13</b>	<b>Problem Solving Methods for Mathematical Modelling</b> . . . . .	173
	Gilbert Greefrath	
<b>14</b>	<b>Improving Mathematical Modelling by Fostering Measurement Sense: An Intervention Study with Pre-service Mathematics Teachers</b> . . . . .	185
	Maike Hagen	
<b>15</b>	<b>How Do Students Share and Refine Models Through Dual Modelling Teaching: The Case of Students Who Do Not Solve Independently</b> . . . . .	195
	Takashi Kawakami, Akihiko Saeki, and Akio Matsuzaki	
<b>16</b>	<b>Exploring Interconnections Between Real-World and Application Tasks: Case Study from Singapore</b> . . . . .	207
	Kit Ee Dawn Ng and Gloria Ann Stillman	
<b>17</b>	<b>Mathematical Modelling Tasks and the Mathematical Thinking of Students</b> . . . . .	219
	Bárbara Palharini and Lourdes Maria Werle de Almeida	
<b>18</b>	<b>Measurement of Area and Volume in an Authentic Context: An Alternative Learning Experience Through Mathematical Modelling</b> . . . . .	229
	Santiago Manuel Rivera Quiroz, Sandra Milena Londoño Orrego, and Carlos Mario Jaramillo López	
<b>19</b>	<b>Mathematical Modelling and Culture: An Empirical Study</b> . . . . .	241
	Jhony Alexander Villa-Ochoa and Mario J. Berrío	

**20 Mathematical Modelling of a Social Problem in Japan:  
The Income and Expenditure of an Electric Power Company . . . . . 251**  
Noboru Yoshimura

**Part III Pedagogical Issues for Teaching and Learning**

**21 The Place of Mathematical Modelling in the System  
of Mathematics Education: Perspective and Prospect . . . . . 265**  
Henry O. Pollak

**22 Moving Within a Mathematical Modelling Map . . . . . 277**  
Rita Borromeo Ferri

**23 Negotiating the Use of Mathematics in a Mathematical  
Modelling Project . . . . . 283**  
Jussara de Loiola Araújo and Ilaine da Silva Campos

**24 Moving Beyond a Single Modelling Activity . . . . . 293**  
Jonas B. Ärlebäck and Helen M. Doerr

**25 The Possibility of Interdisciplinary Integration Through  
Mathematical Modelling of Optical Phenomena . . . . . 305**  
Jennifer Valleriano Barboza, Luana Tais Bassani, Luciano  
Lewandoski Alvarenga, and Lucilaine Goin Abitante

**26 Activation of Student Prior Knowledge to Build Linear  
Models in the Context of Modelling Pre-paid Electricity  
Consumption . . . . . 317**  
José Luis Bossio Vélez, Sandra Milena Londoño Orrego,  
and Carlos Mario Jaramillo López

**27 Mathematical Modellers’ Opinions on Mathematical  
Modelling in Upper Secondary Education . . . . . 327**  
Peter Frejd

**28 Modelling, Education, and the Epistemic Fallacy . . . . . 339**  
Peter Galbraith

**29 Reconsidering the Roles and Characteristics of Models  
in Mathematics Education . . . . . 351**  
Toshikazu Ikeda and Max Stephens

**30 Developing Statistical Numeracy: The Model Must  
Make Sense . . . . . 363**  
Janeen Lamb and Jana Visnovska

**31 Mathematical Modelling and Cognitive Load Theory:  
Approved or Disapproved? . . . . . 375**  
Jacob Perrenet and Bert Zwaneveld

<b>32</b>	<b>Social-critical Dimension of Mathematical Modelling</b> . . . . .	385
	Milton Rosa and Daniel Clark Orey	
<b>33</b>	<b>Pedagogical Actions of Reflective Mathematical Modelling</b> . . . . .	397
	Morgana Scheller, Paula Andrea Grawieski Civiero, and Fátima Peres Zago de Oliveira	
<b>34</b>	<b>Context Categories in Mathematical Modelling in Fundamentals of Calculus Teaching</b> . . . . .	407
	Mara Kessler Ustra and Sandro Rogério Vargas Ustra	
<b>35</b>	<b>Applied Mathematical Problem Solving: Principles for Designing Small Realistic Problems</b> . . . . .	417
	Dag Wedelin and Tom Adawi	
<b>Part IV Influences of Technologies</b>		
<b>36</b>	<b>Visualisation Tactics for Solving Real World Tasks</b> . . . . .	431
	Jill P. Brown	
<b>37</b>	<b>Developing Modelling Competencies Through the Use of Technology</b> . . . . .	443
	Ruth Rodríguez Gallegos and Samantha Quiroz Rivera	
<b>38</b>	<b>Model Analysis with Digital Technology: A “Hybrid Approach”</b> . . . . .	453
	Débora da Silva Soares	
<b>39</b>	<b>Collective Production with Mathematical Modelling in Digital Culture</b> . . . . .	465
	Arlindo José de Souza Júnior, João Frederico da Costa Azevedo Meyer, Deive Barbosa Alves, Fernando da Costa Barbosa, Mário Lucio Alexandre, Douglas Carvalho de Menezes, and Douglas Marin	
<b>Part V Assessment in Schools and Universities</b>		
<b>40</b>	<b>Learners’ Dealing with a Financial Applications-Like Problem in a High-Stakes School-Leaving Mathematics Examination</b> . . . . .	477
	Cyril Julie	
<b>41</b>	<b>Evidence of Reformulation of Situation Models: Modelling Tests Before and After a Modelling Class for Lower Secondary School Students</b> . . . . .	487
	Akio Matsuzaki and Masafumi Kaneko	

<b>Part VI Applicability at Different Levels of Schooling, Vocational Education, and in Tertiary Education</b>	
<b>42 Mathematical Modelling in the Teaching of Statistics in Undergraduate Courses . . . . .</b>	<b>501</b>
Celso Ribeiro Campos, Denise Helena Lombardo Ferreira, Otávio Roberto Jacobini, and Maria Lúcia Lorenzetti Wodewotzki	
<b>43 Models and Modelling in an Integrated Physics and Mathematics Course . . . . .</b>	<b>513</b>
Angeles Domínguez, Jorge de la Garza, and Genaro Zavala	
<b>44 Research-Based Modelling Teaching Activities: A Case of Mathematical Positioning with GNSS . . . . .</b>	<b>523</b>
Xiaojun Duan, Dan Wang, and Mengda Wu	
<b>45 Mathematical Texts in a Mathematical Modelling Learning Environment in Primary School . . . . .</b>	<b>535</b>
Ana Virgínia de Almeida Luna, Elizabeth Gomes Souza, and Larissa Borges de Souza Lima	
<b>46 A Differential Equations Course for Engineers Through Modelling and Technology . . . . .</b>	<b>545</b>
Ruth Rodríguez Gallegos	
<b>47 Contributions of Mathematical Modelling in Education of Youth and Adults . . . . .</b>	<b>557</b>
Jonson Ney Dias da Silva, Taise Sousa Santana, and Carlos Henrique Carneiro	
<b>48 Pre-service Mathematics Teachers' Experiences in Modelling Projects from a Socio-critical Modelling Perspective . . . . .</b>	<b>567</b>
Mónica E. Villarreal, Cristina B. Esteley, and Silvina Smith	
<b>49 A Mathematical Modelling Challenge Program for J.H.S. Students in Japan . . . . .</b>	<b>579</b>
Akira Yanagimoto, Tetsushi Kawasaki, and Noboru Yoshimura	
<b>Part VII Modelling and Applications in the Lived Environment</b>	
<b>50 Modelling the Wall: The Mathematics of the Curves on the Wall of Colégio Arquidiocesano in Ouro Preto . . . . .</b>	<b>593</b>
Daniel Clark Orey and Milton Rosa	
<b>Refereeing Process . . . . .</b>	<b>605</b>
<b>Index . . . . .</b>	<b>607</b>

# Chapter 1

## Cultural, Social, Cognitive and Research Influences on Mathematical Modelling Education

Gloria Ann Stillman, Werner Blum, and Maria Salett Biembengut

**Abstract** This contribution from the ICTMA community on the latest in research and teaching ideas in the area of mathematical modelling and applications education differs from previous volumes in that there is a much stronger emphasis on social and cultural influences on modelling education because of the location of the preceding conference in Brazil. However, another point of difference is the number of chapters that are influenced by cognitive perspectives as there are strong research teams taking this perspective internationally. This chapter situates the work in this volume within the field which has led to much research and evaluative studies in the last decade.

### 1.1 Introduction

According to Niss (2001), the majority of activity in the applications and modelling field in mathematics education was ‘proto-research’ up until the 1990s. Even in the decade from 1990 to 2000, he identified only 50 papers that he considered were genuine research. However, the next decade saw more consolidation of the field as the ICMI study was held and the post conference volume (Blum et al. 2007)

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published. Since that time there has been an increasing number of research studies carried out and quite some diversification of the field. There has been, for example, an increase in the studies that take a cognitive perspective. The publication of this volume following ICTMA 16 in Brazil in 2013 is expected to not only add to this rich research output but also possibly stimulate new lines of inquiry. We hope that it will contribute to inspiring more research into social and cultural influences in the future.

Niss et al. (2007) noted as one imperative of the field of modelling and applications education the articulation of its relationship to the world we live in. This is in pursuit of the goal of “linking of the field of mathematics with some aspects of the world, with the purpose of enhancing knowledge, but also ensuring or advancing the sustainability of health, education and environmental well-being, and the reduction of poverty and disadvantage” (pp. 17–18). Others have attempted to look critically at the role of applications of mathematics and mathematical modelling itself in society. Such sentiments are in keeping with development during education of what is called a socio-critical competency that emphasises the social/political context of modelling (Stillman et al. 2013). It is a feature of this volume that several authors take this emphasis in their work (e.g., Luna, Souza and Lima and Villarreal, Esteley and Smith). Within this context this edited collection represents the current state of thinking and research in this diverse international community distilled and synthesised after the biennial conference. However, it is important that any published contributions stimulated by the conference be connected to the on-going body of research and scholarship in this field as well as previous outputs from the community (e.g., Kaiser et al. 2011; Lesh et al. 2010; Stillman et al. 2013). In this chapter we attempt a first pass on this task.

## 1.2 Innovative Practices in Modelling Education Research and Teaching

Many issues related to the successful solution of real world tasks and problems through mathematical modelling have been identified for some time now in the international literature (see Blum et al. 2007; Niss 2001, for an overview). It seems that in order to resolve these issues we need innovative methods and practices in research into modelling education as well as innovative pedagogical practices. Innovative research methods give us new tools to prise open old problems in order to look at these in a new light. At the same time, innovative classroom practices could bring more teachers into the fold who are able to realise the espoused potential benefits of modelling for students in allowing them to utilise mathematics as a means of “experiencing the world” (Chapman 2009).

A unique Latin American perspective to modelling is brought by the work of Ubi D’Ambrosio. In his chapter he discusses how knowledge is generated (i.e., cognition), how it is individually and socially organised (i.e., epistemology) and how it is

confiscated by power structures, institutionalised and given back to the people who generated it through filters (i.e., politics). How knowledge is built up by individuals and within societal groups is examined. “The full cycle of knowledge includes its generation, individually and socially, its organisation, its expropriation, institutionalization, transmission and diffusion, through systems of education and different forms of filters (such as examinations, degrees, certifications)” (p. 42). Mathematical modelling is seen as a strategy for building up systems of knowledge in different cultural environments. To this end, mathematical modelling allows humans to understand, to explain and to cope with selected facts and phenomena of reality and ideally reality as a whole. Our natural limitations as human beings restrict our access to only selected facts and phenomena (cf Skovsmose 1994). Modelling, by its iterative nature, “allows a better understanding of the selected facts and phenomena of reality, which is the goal that justifies our practices as scientists” (p. 44).

The meaning assigned by students to the problem studied in mathematical modelling activities is considered by Almeida and Silva who adopt a Peircean semiotic (see Sebeok 1991) approach, where meaning is associated with interpretants produced by the students during activity development. The authors highlight the meaning attributed by students to elements of the problem when they attempt modelling activities, through analysing their actions constituted in signs used to suggest and represent the object being dealt with. Thus, Almeida and Silva contend a method to understand the meaning assigned to a problem is to analyse the interpretants of the students during the resolution of that problem as well as what is said about the problem after the signs are generated. A case study of one student’s actions as his group attempts to model tree pruning so as to optimise street lighting is reported to explain this idea in detail. Analyses of the case reveal that familiarity with the real problem for the student and an intention to make meaning from a reference were critical. Almeida and Silva conclude that even though the problem is defined early in the development of a modelling activity, the assignment of meaning is consolidated throughout to the extent that the student defines, interprets and validates a solution. Thus modellers who are successful do not lose sight of the referents that make it meaningful for them in the real world (Nunes et al. 1993; Stillman 2000).

The Models and Modelling Perspective (MMP) (Lesh and Doerr 2003; Lesh and English 2005) for both teaching and researching modelling is now over 30 years old. Brady, Lesh and Sevis describe an innovative research effort to expand the reach of the MMP tradition, engaging questions about the interconnected models and modelling processes of students and teachers at larger, course-length scales. New tools (e.g., Learning Progress Maps and Concept Analysis Wheels) are discussed as well as new directions. Learning progressions using the metaphor of finding your way around a terrain or an evolutionary model are highlighted as possible new research ideas. In addition, further work on the idea of teaching problem solving and heuristics is mooted but reconceptualised within the MMP tradition.

Niss in his contribution makes a distinction between two kinds of mathematical modelling purposes and related modelling endeavours, *descriptive modelling* and