Lecture Notes in Applied and Computational Mechanics 75

Simon R. Eugster

Geometric Continuum Mechanics and Induced Beam Theories

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Preface

In the last centuries continuum mechanics developed from a theory treating very specific problems to a general theory suitable for many applications. Continuum mechanics started with the description of one-dimensional continua where Euler's elastica is maybe its most famous problem. With the seminal work of Cauchy on the existence of the stress tensor in a three-dimensional continuum, the foundations of modern continuum mechanics have been laid down. After a century with the paradigm of infinitesimal deformations and linear elastic material laws, the second half of the twentieth century has been dominated by finite strain theories with large deformations and nonlinear material laws. Especially with the emergence of the computer and its fast rising power, to date, it is possible to treat more complex mechanical behavior than ever before. Nevertheless, an axiomatic consideration of continuum mechanics together with an appropriate mathematical framework is still a major challenge. The foundations of mechanics deal with the identification of the fundamental objects and the postulation of its principles. Due to the high level of abstraction, the mathematical discipline of intrinsic differential geometry seems to be best suited for the description of continuum mechanics. Step-by-step, additional mathematical structure can be introduced and motivated by the underlying physics. Without specifications of constitutive laws, geometric continuum mechanics is on the one hand coordinate independent and on the other hand a priori metric independent. Since a geometric continuum mechanics generalizes the well-established objects of the classical theories, every single object has to be rethought and evaluated if it is fundamental or not.

This book is intended to make the reverse direction of the historical development. It starts with an attempt of geometric continuum mechanics where body and physical space are assumed to be smooth manifolds. Combining the mechanical principles of Paul Germain from the 1970s with an intrinsic differential geometric description of continuum mechanics of Reuven Segev of the 1980s, the principle of virtual work emerges as the fundamental principle of continuum mechanics. In the second part of the book, the classical model of the physical space, the three-dimensional Euclidean space, is assumed and induced beam theories are treated as an application of continuum mechanics. Then it is possible to consider a beam as a continuous body with a constrained position field guaranteed by a perfect constraint stress field. Defining a constrained position field and applying the restricted kinematics to the principle of virtual work of a continuous body, the constraint stresses are eliminated due to the principle of d'Alembert–Lagrange and the weak variational form of an appropriate beam theory is induced directly. This induced approach to beam theory relates the point of view of beams as generalized one-dimensional continua to the theory of continuous bodies. In this work all classical beam theories, in which the cross sections remain rigid and plain, are presented. Additionally, augmented beam theories allowing for cross section deformations are derived using the very same procedure. All theories are suitable for large displacements and large rotations. The obtained weak variational forms of the appropriate beam theories serve then as the basis for the numerical implementation by finite elements.

The work presented in this book has been carried out during my time as research assistant at the Center of Mechanics at the ETH Zurich and appeared as doctoral thesis with the title "On the Foundations of Continuum Mechanics and its Application to Beam Theories". I was accompanied by many people whom I would like to thank for their kind support of my work. I am very thankful to my supervisor Prof. Dr.-Ing. Dr.-Ing. habil. Christoph Glocker for supporting and guiding my research. I have got the opportunity to delve into the very foundations of mechanics and by the way to improve my mathematical background enormously. His distinct idea of mechanics, based on the principle of virtual work as its fundamental principle, has always been a clear guideline to my work. Special thanks go to Prof. Dr. ir. habil. Remco Leine who has taught me the art of academic writing, has been an ideal of how to present research results, and has always been a critical voice in my research. I am looking forward to an intensive time as "Akademischer Rat" working together with him at the Institute for Nonlinear Mechanics at the University of Stuttgart. Many thanks go to Dr.-Ing. O. Papes who hooked me as a student to continuum mechanics and who infiltrated my mind by the concept of a geometric description of continuum mechanics. Finally, I would like to thank my family and friends for their support and continuous encouragement.

Stuttgart, December 2014 Simon R. Eugster

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Part II Induced Beam Theories

Chapter 1
Introduction

This monograph is concerned with fundamental questions on the foundations of continuum mechanics and its application to beam theories. It does not pretend to be in any way 'complete', but merely serves as a discussion about novel approaches applied to these very classical fields of mechanics.

This first chapter starts with a short introduction and motivation for this book. Subsequently, Sect. 1.2 sheds some light on the virtual work in mechanics. After a literature survey in Sect. 1.3, the aim and scope of the work is presented in Sect. 1.4. An outline of the monograph is given in Sect. 1.5.

1.1 Motivation

One of the main goals of mechanics is the description and the prediction of the motion of mechanical devices, machines and mechanical processes. To meet this aim, abstract mechanical theories are formulated, thereby applying concepts from mathematical science. In such a determinism, a strict separation between reality and mathematical abstraction, called the model, has to be considered. The modeling process, being the procedure of the mathematical abstraction, is an interaction between the choice of the assumed mathematical structure and the description of observations in the real world within this mathematical framework. Hence, a mechanical theory can be developed on different levels of mathematical abstraction. The higher the level of mathematical abstraction, the less mathematical objects are involved and the more general a mechanical theory is. By increasing the level of abstraction in a mechanical theory, we try to extract the essential mechanical objects. An important step on that route of abstraction is the description of mechanics with as little mathematical structure as necessary, to recognize the fundamental laws of mechanics. There exists a vast amount of specific mechanical theories in which many assumptions on the kinematics of the system and on constitutive level are taken. For instance, we may distinguish between rigid body mechanics, beam theories, shell

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theories, theory of elasticity, theory of fluids, finite degree of freedom mechanics to name a few. A fundamental question emerges: does a mechanical theory on a high level of abstraction exist which is able to induce these specific theories? The question immediately asks for the assumptions and concepts to arrive in a rigorous way at all these specific theories. The question of the embedding of well-known specific theories in a more general mechanical theory is one of the major challenges of modern classical mechanics. Such an embedding of theories leads to a more compact formulation of the vast field of classical mechanics. It leads to a deeper understanding of mechanics and will eventually allow treating more complex mechanical systems. This is what can be understood as scientific progress.

1.2 The Virtual Work

A rather novel insight in analytical mechanics is that the virtual work of a mechanical system is invariant with respect to the change of coordinates. This is directly related to the fact that there is a (coordinate free) differential geometric definition of the virtual work. To explain the basic idea, consider the case of finite degree of freedom mechanics, where the configuration manifold fully describes the kinematic state of the mechanical system. A generalized virtual displacement is a tangent vector of the configuration manifold. A covector of the configuration manifold as an element of the cotangent space constitutes a generalized force. The virtual work is defined as the real number obtained by the evaluation of a generalized force acting on a generalized virtual displacement. This geometric definition of the virtual work is completely free of any choice of coordinates and does not require any further geometric structure such as a metric. With the geometrical point of view in mind, the determination of the configuration manifold, being the kinematic description of the mechanical system, induces the space of generalized forces of the mechanical system. In a nutshell, the choice of kinematics defines, in the sense of duality, what kind of forces we may expect.

An illustrating example is a moving particle in the Euclidean three-space, where the very same space corresponds to the configuration manifold of the particle. Consequently, the generalized forces are elements of the cotangent space of the Euclidean three-space. In the Euclidean three-space there exist two important isomorphisms. One isomorphism is a canonical isomorphism between the tangent space and the Euclidean three-space. The second isomorphism is the isomorphism between tangent and cotangent space induced by the Euclidean metric. Using both isomorphisms, a generalized force on the particle can be identified with an element from the Euclidean three-space. This corresponds to the very classical understanding of a force as a geometric object from the Euclidean three-space satisfying the parallelogram law. As a side note, it is meaningless to speak of such thing as a couple of the particle, since there is no kinematic counterpart in the description of a particle.

Being the invariant object in mechanics, the virtual work almost naturally emerges as a central element in the postulation of the fundamental laws of mechanics.

The virtual work of a mechanical system is the sum of the virtual work contributions of all forces of the mechanical system. The principle of virtual work, stated as an axiom, claims that the virtual work of a mechanical system has to vanish for all virtual displacements. Hence, the principle of virtual work as a fundamental mechanical law is a coordinate free and metric independent formulation. Introducing more geometric structure as e.g. a metric, it is possible to formulate constitutive laws which relate force quantities with kinematic quantities and to arrive at more specific mechanical theories. For instance, a metric is required to define the strain of a continuous body which is necessary for the formulation of a material law. Another example is the formulation of the dynamics of a particle moving in the Euclidean three-space. The linear relation between the velocity of the particle and the linear momentum needs a metric of the space and the mass of the particle as a proportionality factor. Thus, from a differential geometric point of view, the linear momentum as "as mass times velocity" can be considered as an assumption on constitutive level.

In computational mechanics for infinite dimensional systems, the principle of virtual work in the form of weak variational forms is a fully accepted concept. It is used to perform existence and uniqueness proofs on the one hand, and to develop numerical schemes on the other hand. As a variational formulation, the principle of virtual work provides the only possibility within classical mechanics to mathematically define perfect bilateral constraints. The latter is done in form of a variational equality, known as the principle of d'Alembert–Lagrange, which puts the constraint forces into the annihilator space of the admissible virtual displacements. The concept of perfect constraints is omnipresent in each branch of mechanics and is quintessential to induce more specific theories from a general mechanical theory.

Many specific mechanical theories can be considered as special cases of the theory of continuous bodies. Rigid body mechanics, for instance, is the dynamics of a continuous body whose deformation is constrained such that the position field of the body can be described by a displacement of one material point of the body and a rotation of the body. Hence, the rigid body can be considered as a constrained continuous body. As discussed above, the principle of virtual work as a variational formulation is the only way to treat perfect bilateral constraints. Consequently, to induce a specific mechanical theory from the theory of a continuous body by imposing further constraints on the mechanical system, a variational formulation of the dynamics of a continuous body is inevitable.

In order to obtain an intrinsic theory of a continuous body in variational form, we have to use the concepts of analytical mechanics, where the forces are induced by the choice of the kinematics of the mechanical system. Before starting with a play, the actors and the scene have to be determined. Here, the body plays the role of a single actor and the scene is given by the model of the physical space. The play, i.e. how the body performs on the scene, corresponds to the admissible configurations of the body in the physical space. Using appropriate definitions of the body and the physical space, the set of all maps of the body into the physical space build an infinite dimensional manifold, called configuration manifold. This configuration manifold induces as in the finite dimensional setting the space of forces in the sense of duality. Applying the principle of virtual work together with further assumptions,