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ADVANCES IN MECHANICS AND MATHEMATICS 21

Mechanics of Generalized Continua

One Hundred Years After the Cosserats

Springer

MECHANICS OF GENERALIZED CONTINUA

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ONE HUNDRED YEARS AFTER THE COSSERATS

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Foreword

Welcome!

I am especially privileged and honored that Professors Maugin, Metrikine and Erofeyev, the organizers and chairmen of this meeting, the Euromech Colloquium 510 honoring the Cosserats for the 100 year anniversary of the publication of their book, have asked me to say a few words to express my welcome salute to you. Much as I would have liked to do this in person, my physical being is no longer keeping pace with my mental desires and, thus, alas, is denying me this luxury.

Sometime in the past, I remember reading an article whose author's name has slipped my memory—perhaps it was Marston Morse, Professor Emeritus at the Institute for Advanced Study, who wrote (and I paraphrase):

Discovery of new mathematical disciplines originates from two criteria:

1. **Generalization**
2. **Inversion**

Some of the earliest examples for the validity of these criteria are:

(a) The Newton–Leibniz discovery of differentiation and integration, which started calculus; and
(b) The Theory of Elasticity, which was conceived when Robert Hooke, in 1678, published an anagram: "ceiiinosssttuu", which he expressed as "*ut tensio sic vis*", meaning, the power of any material is in the same proportion within the tension thereof. Presently, this is known as "Hooke's Law".

Some 250 years later, "The modern theory of elasticity may be considered to have its birth in 1821, when Navier first gave the equations for the equilibrium and motion of elastic solids, . . . " (Todhunter and Pearson).

Of course, many other scientists, Cauchy, Poisson, Stokes, and others, after 1821, improved and extended the theory to other materials, e.g., viscous fluids, and they investigated atomic and molecular foundations. This is typical—for the maturation of any discipline is the result of the contributions of many scientists and often takes a long time.

Improvements and extensions of the theory of elasticity continued in the nineteenth and early part of the twentieth century: rigorous mathematical theory of nonlinear elasticity, relativistic continuum mechanics, magneto-elasticity and other "hyphenated" sister fields, like viscoelasticity and thermoelasticity. Underlying basic postulates (e.g., frame-independence, thermodynamical restrictions, relativistic invariance) were introduced and applied in the development of field equations and admissible constitutive laws. Research in granular and porous elastic solids, composite elastic materials, polymeric materials, and statistical and molecular foundations of continua are but a few examples that still remain as active research fields.

Eugène Maurice Pierre Cosserat and his brother François Cosserat, 100 years ago, cast the seed of **Generalized Continua**, by publishing a book, in 1909, entitled *Théorie des Corps Déformables* (Hermann, Paris). The revolutionary contribution of this book is that material points of an elastic solid are considered equipped with directors, which give rise to the concept of couple stress and a new conservation law for the moment of momentum. By means of a variational principle which they called "***l'action euclidienne***", they obtained "balance laws of elasticity". The introduction of the director concept made it possible to formulate anisotropic fluids, e.g., liquid crystals, blood.

The Cosserats did not give constitutive equations. These, and the introduction of the microinertia tensor and the associated conservation law, which are crucial to the dynamic problems in solid and fluent media (e.g., liquid crystals, suspensions, etc.) were introduced later by other scientists.

Over half a century elapsed before the Cosserats' book was discovered by researchers. After 1960, independent, Cosserat-like theories were published in European countries, the USA and the USSR, under a variety of nomenclature (e.g., couple stress, polar elasticity, asymmetric elasticity, strain gradient theories, micropolar elasticity, multipolar theory, relativistic continua with directors, etc.). I recall a literature search on these subjects that was shown to me by a visiting scholar, Professor Listrov, from the USSR This book contained several hundred entries of papers published by 1970.

The next significant generalizations appear in 1964 and thereafter, in the areas of microelasticity, microfluid mechanics, micropolar continua, micromorphic electrodynamics, and others that constitute the family of micromorphic continua or microstructure theories.

The conception of these theories was based on the query, "Is it possible to construct continuum theories that can predict physical phenomena on the atomic, molecular, or nano scales?" These would require supplying additional degrees of freedom to the material point beyond a director. After all, the molecules that constitute the internal structures of the material points (particles) undergo deformations and rotations arising from the displacement and rotations of their constituent atoms. This supplies twelve degrees of freedom. A body with such an internal structure is called **Micromorphic grade 1**. Micromorphic continua of grade $N > 1$ have also been formulated.

To understand the difference between the Cosserat and the micromorphic elasticities, it is important to note that micromorphic elasticity gives rise to two different second-order strain tensors (only one of which is symmetric), and to one third-order microstrain tensor. Correspondingly, the balance laws introduce two second-order stress tensors (only one of which is symmetric), and one third-order microstress (moment-stress) tensor.

In special cases, the Micromorphic Theory leads to other special continuum theories:

Micromorphic → Microstretch → Micropolar (Cosserats) → Classical

The next important contributions are the nonlocal continuum theories that generalize constitutive equations for classical and micromorphic continuum theories, by introducing the influence of distant material points, e.g., the stress tensor is a functional of the strain tensors of all material points of a body. In this sense, micromorphic grade 1 is a nonlocal theory with a *short nonlocality* (or discrete nonlocality). Among the many important contributions of nonlocality, I mention that it eliminates the stress singularity (infinite stress) at the crack tip predicted by classical elasticity. Moreover, a natural fracture criterion was born which states that failure occurs when the maximum stress becomes or exceeds the cohesive stress.

The Present State. No doubt other generalized continuum theories are in a state of composition. But mathematical theories cannot be considered the truth without experimental verification. Unfortunately, excluding classical theories, the experimental work for all these theories is left wanting. The opportunity is here and now, for experimentalists to determine the material moduli and/or to confirm or challenge the validity of some of these theories.

A Note on the Future. Ultimately, all continuum theories must be based on the quantum field theory, or perhaps, on the quantum theory of general relativity (when unified). This offers the greatest challenges to future scientific investigators.

I am pleased to see so many interesting contributions to some of these fields included in this meeting, which are in the spirit of the Cosserats' work.

I welcome you and send my best wishes for what, I am sure, will be an inspirational and productive meeting.

Littleton, Colorado, May 2009

A. Cemal Eringen
Professor Emeritus,
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Preface

This volume gathers in some organized and edited manner most of the contributions delivered at the EUROMECH Colloquium 510 held in Paris, May 13–16, 2009. The explicit aim of the colloquium was, on the occasion of the centennial of the publication of a celebrated book (Théorie des corps déformables) by the Cosserat brothers, to examine the evolution in time since the Cosserats, and the actuality of the notion of generalized continuum mechanics to which the Cosserats' work contributed to some important extent. Of course, the Cosserat book belongs to this collection of classics that are more often cited than read. The reason for this is twofold. First, the vocabulary and mathematical symbols have tremendously evolved since the early 1900s, and second, the Cosserat book by itself is an intrinsically difficult reading. As a matter of fact, more than introducing precisely the notion of Cosserat media (a special class of generalized continua), the Cosserats' book had a wider ambition, that of presenting a reflection on the general framework of continuum mechanics, with the notion of group permeating—not explicitly—its structure (cf. the notion of "action euclidienne"). This is reflected in many of the following contributions.

Overall, the whole landscape of contemporary generalized continuum mechanics was spanned from models to applications to structures, dynamical properties, problems with measurement of new material coefficients, numerical questions posed by the microstructure, and new possible developments (nanomaterials, fractal structures, new geometrical ideas). Remarkably absent were models and approaches using the concept of strong nonlocality (constitutive equations that are functionals over space). This is a mark of a certain evolution.

An interesting comparison can be made with the contents of the landmark *IUTAM Symposium* gathered in 1967 in Stuttgart-Freudenstadt under the chairmanship of the late E. Kröner. Most of the models presented at that meeting by luminaries such as Noll, Eringen, Rivlin, Green, Sedov, Mindlin, Nowacki, Stojanovic, and others were essentially of the Cosserat type and, still in their infancy, had a much questioned usefulness that is no longer pondered. Most of the contributions were either American or German. With the present EUROMECH we witnessed an enlargement of the classes of models with a marked interest in gradient-type theories. Also, because the political situation has drastically changed within forty tears, we realize now the importance of the Russian school. The latter was, in fact, very much ignored in the 1960s and 1970s while some Russian teams were ahead of their Western colleagues in acknowledging their debt to the Cosserats and other scientists such as Leroux, Le Corre and Laval in France. Of these heroic Soviet times, E. Aero and V. Palmov, both from St. Petersburg, who published on the subject matter in the early 1960s, were present in Paris. Professor A.C. Eringen (he also in Freundenstadt in 1967), unable to attend, kindly sent us a Welcome address that is reproduced here in the way of a Foreword.

Unfortunately, the editing of this book was saddened by the passing away of A.C. Eringen on December 06, 2010, at the age of 88, after more than sixty years of devotion to engineering science, physics and applied mathematics.

The Colloquium was financially and materially supported by the Engineering UFR of the *Université Pierre et Marie Curie* (UPMC), the STII Directorate of the French *Centre National de la Recherche Scientifique*, and the *Institut Jean Le Rond d'Alembert*, UPMC–Paris Universitas and UMR 7190 of CNRS. Members of the MPIA Team of this Institute helped much in the local organization. Ms Simona Otarasanu is to be thanked for her efficient treatment of many questions. Without the expertise of Ms Janine Indeau, the present volume would not exist.

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