Chapter 2
Product Design in a Market with Satisficing Customers

Matulya Bansal and Costis Maglaras

Abstract We study the product design problem of a revenue-maximizing firm that serves a market where customers are heterogeneous with respect to their valuations and desire for a quality attribute and are characterized by a perhaps novel model of customer choice behavior. Specifically, instead of optimizing the net utility that results from an appropriate combination of prices and quality levels, customers are "satisficers" in that they seek to buy the cheapest product with quality above a certain customer-specific threshold. This model dates back to Simon's work in the 1950s and can be thought of as a model of bounded rationality for customer choice. We characterize the structural properties of the optimal product menu for this model and explore several examples where such preferences may arise.

2.1 Introduction

How do consumers trade off price to quality of service in choosing a product among various substitutable alternatives offered by the same or by competing firms? As a concrete example, how do users tradeoff speed of an Internet service connection with the price they have to pay? How should a firm design its product menu to optimize its profitability taking into account the strategic consumer choice behavior? The answers to these questions depend crucially on our understanding of how consumers perceive delay (or, more generally, product quality), their degree of heterogeneity in terms of delay sensitivity, and value for the offered product and on how delay costs and the prices of the various product options are combined and

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used in making a product choice decision. This chapter addresses the above questions under a novel model of choice behavior, where consumers rather than being utility maximizers of some sort are "satisficers" in that they seek to buy the cheapest product with quality above a certain consumer-specific threshold. This model dates back to Simon's work in the 1950s (Simon 1955, 1956).

As a running example we will consider a service provider (SP) offering a product, such as an ISP connection or software-on-demand that is susceptible to congestion effects and, therefore, delays; we use the terms consumer, customer, and, at times, user, interchangeably. Potential customers are heterogeneous in their valuations and delay sensitivities. Expected delay here captures the notion of quality, with lower delay implying higher quality. The SP's problem is to select a menu of product variants that are defined through their price and associated delay that maximizes its expected profits. A classical model of customer choice behavior for this problem is due to Mendelson and Whang (1990) which postulates that a customer with valuation v for the offered service enjoys a net utility of $u_i = v - (p_i + c(d_i))$ from the ith variant that is priced at p_i and has an associated delay of d_i time units and where $c(\cdot)$ is a customer-specific delay cost function expressed in \$ per unit of delay. Mendelson and Whang (1990) used a linear delay cost function of the form $c(d_i) = c \cdot d_i$, Dewan and Mendelson (1990) use a delay cost of the form c(d) = $c \cdot (d - \theta)^+$, van Mieghem (1995) allowed for general, convex increasing delay functions, while Ata and Olsen (2008) introduced delay functions that are convex increasing and then become concave increasing after a point. Figure 2.1 shows the linear, quadratic, piecewise-linear (convex increasing), and the proposed cost function vs. delay. Given a set of product variants characterized by (p_i, d_i) a customer will select product

$$i^* = \arg\max_i \; \{v - (p_i + c(d_i)) \; : \; v \geq p_i + c(d_i)\}. \tag{2.1}$$

Another alternative model that is close to the vertical differentiation literature could postulate that the net utility associated with product variant i is $u_i = (v - p_i)g(d_i)$,

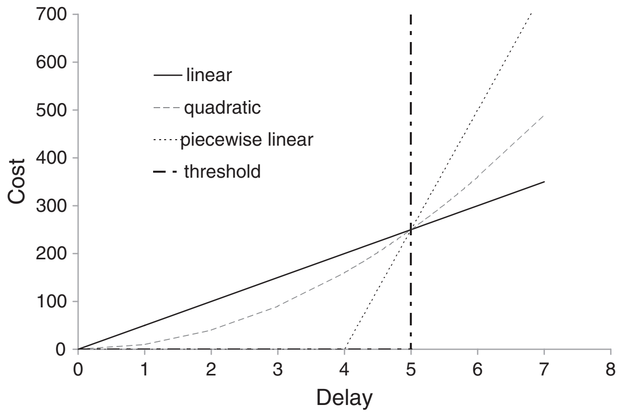

Fig. 2.1 The linear, quadratic, piecewise-linear, and proposed cost functions vs. delay.

where $g(\cdot)$ is a multiplicative factor whose magnitude depends on the quality of the offered product. Again, each customer would choose the product variant i that would maximize the resulting net utility.

Most of the above papers have focused on social welfare optimization as opposed to revenue maximization that is of interest in this chapter; in the cases where the revenue maximization objective has been considered, the emphasis has been on markets with two types of customers and product menus with two variants, Afeche (2005) and Maglaras and Zeevi (2006). The multi-type problem is hard, even in a deterministic setting, and, perhaps more importantly, assumes that the utility maximizing customers are solving an intricate problem in order to make their choice decision that in turn affects crucially the seller's product design decision.

In some practical settings it might be more realistic to assume that customers only care whether or not the product quality lies above a customer-specific threshold, and not by how much; e.g., video conferencing is associated with a bandwidth requirement, but additional bandwidth above that level is not necessarily beneficial. In those settings it might be appealing to assume that customer preferences with respect to the quality attribute are "dichotomous" such that all products with quality at least as good as a customer-specific quality threshold are acceptable to the customer, while all products with quality below the customer-specific threshold are unacceptable. From then on, among the acceptable products, if any, the customer buys the cheapest one, provided the price of this product does not exceed customer valuation.

An alternative motivation could be that the dichotomous decision rule is a simplification of the fully rational decision described earlier that is based on net utility calculations and serves as a "bounded rationality" surrogate for the potentially complex decision rule embodied in (2.1) or similar variants to it.

The baseline model that we will consider herein is that of a firm selling a good or service in a market of heterogeneous customers. The good or service is characterized by a one-dimensional quality attribute, such as delay, and to maximize revenues, the firm seeks to discriminate customers by creating multiple qualities and offering them at different prices. We assume that differentiation does not entail any cost. The firm offers M products, with p_j and q_j denoting, respectively, the price and quality of product j, $j = 1,\ldots,M$. The capacity available to the firm is denoted by C. We assume that there are N customer types that are segmented according to their quality preferences. Each type i customer has a valuation v_i for the product, which is an independent draw from a general distribution $F(\cdot)$ with support $[0,\bar{v}_i]$, and a strictly positive density $f_i(\cdot)$ on $[0,\bar{v}_i]$, and, a quality threshold θ_i, such that he or she is only willing to purchase product variants j whose quality q_j is at least as large as θ_i, i.e., $q_j \geq \theta_i$. The quality threshold is common across all type i customers. The size of the type i market segment is denoted by Λ_i. We assume that types are labeled in such a way that $\infty > \overline{\theta} > \theta_1 > \theta_2 > \cdots > \theta_N > \underline{\theta} > 0$, with a higher value implying the desire for a better quality, and $\infty \geq \bar{v}_1 \geq \bar{v}_2 \geq \cdots \geq \bar{v}_N > 0$, i.e., customers having higher quality thresholds have a maximum valuation at least as high as the maximum valuation of customers having lower quality thresholds. Let $\bar{F}_i(\cdot) = 1 - F(\cdot)$ be the complementary cumulative distribution function for type i

valuations. We will assume that $\lim_{p\to\infty} p\overline{F}_i(p) = 0$, $i = 1,\ldots,N$, i.e., the revenue from any customer type goes to 0 as the price goes to infinity (this holds trivially for class i if $\overline{v}_i < \infty$).

Customers are satisficers in that they strictly prefer the cheapest product whose quality exceeds their respective quality threshold and purchase that product if their valuation exceeds its price. In more detail all type i customers prefer product $\chi_i(p,q)$ given by

$$\chi_i(p,q) = \begin{cases} \arg\min p_j, & \exists\, q_j \geq \theta_i, \\ 0, & \text{otherwise}, \end{cases} \tag{2.2}$$

where p_j and q_j denote, respectively, the price and the quality of the jth product offered. If $\chi_i(p,q) = l,\;\; l \geq 1$, the demand from type i customers for this product is given by $\Lambda_i \overline{F}_i(p_l)$, and the revenue by $p_l \Lambda_i \overline{F}_i(p_l)$. If $\chi_i(p,q) = 0$, then type i customers do not find any product from the firm to be acceptable in terms of their quality.

The firm's revenue maximization problem is to choose the number of product variants to offer, M, as well as the corresponding prices and quality levels p_j, q_j for $j = 1,\ldots,M$ to solve the following problem:

$$\max_{p,q,M} \sum_{j=1}^{M} p_j \left[\sum_{i=1}^{N} \Lambda_i\, \overline{F}_i\,(p_j)\; 1_{\{\chi_i(p,q)=j\}} \right] \tag{2.3}$$

$$\text{s.t.} \sum_{j=1}^{M} \sum_{i=1}^{N} \Lambda_i\, \overline{F}_i\,(p_j)\; 1_{\{\chi_i(p,q)=j\}} \leq C, \tag{2.4}$$

$$0 \leq p < \infty, \;\; 0 \leq q < \infty, \tag{2.5}$$

$$1 \leq M < \infty, \quad M \text{ integer}. \tag{2.6}$$

The objective in (2.3) is the sum of revenues across the M products, where revenue for product j equals the price of the product multiplied by the number of customers that buy it. Equation (2.4) restricts the volume sold across customer types to be less than or equal to the available capacity C. For convenience, we have assumed that there exists a product 0 (corresponding to the case that customers do not find any product from the firm to be acceptable) and set $p_0 = 0$ and $q_0 = 0$.

This chapter lists several possible applications of this choice behavior for problems of practical interest, study the above mathematical problem, and sketch out several extensions. Our treatment is not exhaustive, but rather tries to highlight some structural result that hinges on this novel choice model, and hopefully motivates further work.

Satisficing is well known in the marketing and psychology literature, see, e.g., Iyengar (2006), and Schwartz et al. (2002), but seems to be novel in the context of the revenue management and operations management literatures. First, satisficing choice behavior can be the result of utility maximizing behavior in settings where the offered service and its anticipated usage are such that the disutility due to quality degradation is essentially flat until a certain threshold is reached and grows at a very rapid ("infinite") rate above that threshold. Perhaps more importantly, this threshold model of customer choice behavior can be motivated as an example of the "simple payoff" function as discussed in Simon (1955). Alternatively, this functional form can be motivated as the limiting case of the S-shaped utility functions, discussed, for example, in Kahneman and Tversky (1979) and Maggi (2004). For example, a utility function that would approximate $w(q)$ is the exponential S-shaped utility function

$$\tilde{w}(q) = \begin{cases} \frac{1}{\beta} + \frac{\beta - 1}{\beta}(1 - e^{-\alpha(q-\overline{q})}), & \text{if } q \geq \overline{q}, \\ \frac{1}{\beta} e^{-\alpha(\overline{q}-q)}, & \text{if } q < \overline{q}, \end{cases} \tag{2.7}$$

where $\alpha > 0$, $\beta \geq 1$, with the approximation becoming exact when $\beta = 1$ and $\alpha = \infty$. This is illustrated in Figure 2.2.

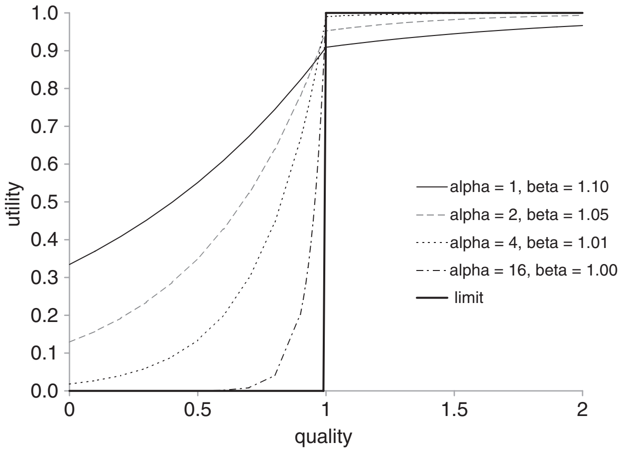

Fig. 2.2 This figure shows how threshold preferences arise as the limiting case of S-shaped utility functions discussed in prospect theory.

The satisficing model can be viewed as a limiting case of the vertical and horizontal differentiation models. In the context of vertical differentiation, satisficing choice behavior corresponds to using the function $g(q_i;\theta) = 1$ if $q_i \geq \theta$, and $g(q_i) = 0$, otherwise; our model would require the $g(\cdot)$ function to be type dependent, which is itself a slight extension of the vertical differentiation literature. In the context of horizontal differentiation, suppose customers differ in their preferences over a single-dimensional quality attribute θ in $[\underline{\theta}, \overline{\theta}]$. Under the traditional model of horizontal differentiation (Hotelling 1929), the quality cost $c(q)$ associated with a product of quality q to a customer with preference θ is $c(q) = t_1(\theta - q)$ if $q < \theta$, and $c(q) = t_2(q - \theta)$, otherwise, where the transportation costs t_1 and t_2 are typically assumed to be the same. Customers again find products that result in a non-negative utility acceptable and maximize their utility over acceptable products. In the limiting, asymmetric case where $t_1 = \infty$ and $t_2 = 0$, the horizontal model of customer choice behavior reduces to the threshold model of customer preferences.

The remainder of this chapter is organized as follows: Section 2.2 concludes with a brief literature review. Section 2.3 presents several examples where modeling customer behavior via threshold preferences is appealing. Section 2.4 characterizes the structure of the optimal solution to the product design problem. Section 2.5 discusses three extensions to the original model and Section 2.6 offers some concluding remarks.

2.2 Literature Survey

Our work builds upon several different areas of revenue management. The primary motivation for our work stems from the interface between marketing, psychology, and prospect theory focusing on customer behavior models. In his classic papers (1955; 1956), Simon questioned the pervasive assumption of agent rationality made in economic models. Citing constraints on information availability and computational capacities of individuals, (1955) Simon proposed “simple payoff functions” such as the one considered in this chapter as an approximation to model complex agent utility. Simon (1956) introduced the idea of “satisficing” to model the behavior of an organism facing multiple goals. In more recent research in psychology, researchers distinguish between “maximizers” and “satisficers,” as discussed in Iyengar (2006) and Schwartz et al. (2002). Wieczorkowska and Burnstein (2004) refer to individuals exhibiting satisficing behavior as adopting an “interval” strategy as opposed to a “point” strategy (maximizing). Schwartz et al. (2002) mention that indeed individuals might not be maximizers or satisficers along all dimensions. In our case, customers satisfice with respect to quality while they maximize with respect to price.

In their famous paper Kahneman and Tversky (1979) propose that individual utility is concave for gains, while being convex for losses. Such utility functions are discussed in Maggi (2004). As discussed earlier, our utility function for quality attribute can be thought of as the limiting case for the S-shaped exponential utility function discussed here. The deadline delay cost structure discussed in Dewan and Mendelson (1990) prescribes zero cost for delay below a certain delay threshold and linear delay costs thereafter. Our delay cost function, like Dewan and Mendelson (1990), posits a zero cost for delay below a customer delay threshold and infinite (or large enough to deter customer from buying this product) costs thereafter.

The second stream of literature that is related to our work studies the second-degree price discrimination problem by a monopolist facing customers that differ in their preference for a quality attribute. Two classic papers are due to Mussa and Rosen (1978) and Moorthy (1984). In Mussa and Rosen (1978), customer utility is linear in quality, and quality is continuous. In Moorthy (1984), customer utility is allowed to be non-linear, but quality is discrete. In both cases, customers are maximizers. Both Moorthy (1984) and Mussa and Rosen (1978) discuss strategic degradation of quality by the monopolist to maximize revenues. This idea of intentionally degrading product quality when offering a product to less quality-sensitive customers so as to achieve differentiation is well known and also discussed in Afeche (2005) and Shapiro and Varian (1998) among other places. In addition to the above papers that discuss the product design problem under vertical differentiation, the product design problem has also been discussed under horizontal differentiation, e.g., Hotelling (1929) and Salop (1979). Duopoly models of product differentiation are considered in Moorthy (1988), Shaked and Sutton (1982), Wauthy (1996), etc. We also study the product design problem under threshold preferences to simultaneous and sequential duopoly models of market entry.

Each of the various application areas that we briefly touch upon later on has a potentially extensive literature that we will not review in this chapter in much detail, but rather simply offer a few passing references. In the area of revenue maximization for queues, in addition to the above-mentioned papers, we also highlight Katta and Sethuraman. There is a fast-growing literature in revenue management that considers the strategic consumer choice behavior, e.g., in deciding when to purchase a product in anticipation of the dynamic price path and associated rationing risk adopted by the seller. In this area we refer the reader to the review article by Shen and Su (2007), Liu and van Ryzin (2005), Su (2007), Cachon and Swinney (2007), and Bansal and Maglaras (2008). All of these assume a fully rational model of customer behavior. This chapter proposes a satisficing model of consumer choice behavior for this problem. Versioning of information goods has been studied in Bhargava and Choudhary (2004) and Ghose and Sundararajan (2005), while Shapiro and Varian (1998) presents several examples of versioning of information goods. Several researchers have addressed the problem of identifying the optimal inventory policy in the presence of multiple demand streams that differ in their tolerance for the minimum fillrate or the maximum leadtime they are willing to accept. Such a specification of acceptable quality levels closely mirrors our model of threshold-based preferences and is considered, for example, in Klejin and Dekker (1998). Finally, Kim and Chajjed (2002) study the product design problem of a monopolist firm offering a product with multiple quality attributes to a market of customers under the classic model of customer choice. The market consists of two customer segments, so at most two products need to be offered. We briefly discuss the extension of our model to the case of multiple quality attributes under some lexicographic ordering.

2.3 Applications and Variations to Basic Model

This section presents a non-exhaustive list of instances of product design problems where the study of customer satisficing behavior may be natural from a practical viewpoint.

2.3.1 Delay Differentiation

In the introduction we briefly reviewed an application of the proposed approach toward the problem of revenue maximization for a service that is susceptible to congestion effects and delay. Our model in this setting is based on a deterministic relaxation that disregards the equations that govern the steady state behavior of the queueing facility that is offering that service; this relaxation can be justified in an asymptotic setting where the market size of processing capacity of the system grows large along the lines of Maglaras and Zeevi (2006).

Delay sensitivity can also arise in other contexts, such as in retailing for fashion goods, where customers may be sensitive as to the time until which they wish to purchase the product; e.g., upon its introduction, in the middle of the regular selling season, after the season has ended. The resulting formulation is identical to the one discussed in the introduction.

2.3.2 Capacity Differentiation

There are several applications where the quality attribute corresponds to the capacity allocated to a customer, such as in the case of an Internet service provider (ISP) that offers bandwidth to domestic and business "end-users." Customers are heterogeneous in their valuations and have threshold preferences with respect to capacity, i.e., the minimum bandwidth they require. There are N customer classes, with class i customer valuations distributed as $F_i(\cdot)$ and class i having a capacity threshold θ_i, the minimum capacity that they desire. We assume that $\theta_1 > \theta_2 > \cdots > \theta_N$. Then denoting as c_j the capacity associated with product j offered by the firm, class i customers seek to buy the cheapest product j such that $c_j \geq \theta_i$. The firm's optimization problem can be stated as follows:

$$\max_{p,c,M} \sum_{i=1}^{N} \sum_{l=1}^{M} p_i \, \Lambda_i \, \overline{F}_i \, (p_i) \, 1_{\{\chi_i(p,c)=l\}} \tag{2.8}$$

$$\text{s.t.} \sum_{i=1}^{N} \sum_{l=1}^{M} \Lambda_i \, \overline{F}_i \, (p_i) c_i 1_{\{\chi_i(p,c)=l\}} \leq C, \tag{2.9}$$

$$0 \leq p < \infty, \quad 0 \leq c < \infty, \tag{2.10}$$

$$1 \leq M < \infty, \quad M \text{ integer.} \tag{2.11}$$

Equations (2.8), (2.10), and (2.11) are analogous to (2.3), (2.5), and (2.6) in the general problem, where c_i now denotes the quality of product i. Note that the capacity allocations c_i enter the seller's capacity constraint (2.9) in a way that is different than in the problem formulated in the introduction and potentially problematic due to the product terms $\overline{F}_i(p_i)\, c_i$; we show later on that due to the structure of the above problem, the capacity constraint simplifies and retains its tractability.

2.3.3 Rationing Risk Differentiation

Consider a monopolist firm that seeks to sell a homogeneous product to a market of heterogeneous, strategic customers that vary in their valuations and degree of risk aversion and where the firm seeks to discriminate its customers by creating rationing risk over time, i.e., by offering the product at different prices and fillrates at different times over the selling horizon; see, e.g., Liu and van Ryzin (2005) for the case where risk preferences are homogeneous and Bansal and Maglaras (2008) for a model where risk preferences may vary across customer types; both of these papers considered utility maximizing choice behavior. With satisficing behavior, customers will a threshold that corresponds to the minimum acceptable fillrate that they are willing to accept. Customers are strategic, observe (or know) the entire pricing and rationing risk trajectories used by the seller, and accordingly make the optimal timing decision to enter the market and purchase the product. The firm's product design problem is to identify the optimal number of products to offer to this market, along with their prices and fillrates. Fillrates r satisfy $0 \leq r \leq 1$, and a fillrate of r implies that only a proportion r of customer requests are fulfilled by the firm. Fillrates here correspond to our notion of quality, with a higher fillrate implying a better quality. There are N types and type i customers have a fillrate threshold θ_i, implying that type i customers prefer the cheapest product j with fillrate $r_j > \theta_i$ (notice we assume that the inequality is strict). We assume $1 > \theta_1 > \theta_2 > \cdots > \theta_N > 0$.

One possible way to motivate such choice behavior is by assuming that customers have a limit on the relative payoff variability they are willing to tolerate. The expected payoff to a customer with valuation v upon deciding to purchase a product with price p and fillrate r is given by $(v-p)r$ and the variance of this payoff is given by $(v-p)^2 r(1-r)$. Let A denote the customer threshold for the variability the customer is willing to tolerate. Hence this customer would seek to purchase the cheapest product such that

$$\frac{\text{stdev}}{\text{mean}} = \frac{\sqrt{(v-p)^2 r(1-r)}}{(v-p)r} \leq A, \tag{2.12}$$

where A is a fixed fraction. This reduces to $r \geq 1/(1+A^2)$, implying that customer has threshold preferences with respect to the rationing risk where the rationing threshold is given by $1/(1+A^2)$. Also, a low desire for variability leads to a higher rationing threshold, which is intuitive.

The optimization problem that the firm faces can be expressed as follows:

$$\max_{p,r,M} \sum_{i=1}^{N} \sum_{l=1}^{M} p_i \, \Lambda_i \, \overline{F}_i \, (p_i) \, r_i \, 1_{\{\chi_i(p,r)=l\}} \tag{2.13}$$

$$\text{s.t.} \sum_{i=1}^{N} \sum_{l=1}^{M} \Lambda_i \, \overline{F}_i \, (p_i) \, r_i \, 1_{\{\chi_i(p,r)=l\}} \leq C, \tag{2.14}$$

$$0 \le p < \infty, \quad 0 \le r \le 1, \tag{2.15}$$
$$1 \le M < \infty, \quad M \text{ integer.} \tag{2.16}$$

The objective (2.13) is the sum of revenue over the N classes, where class i revenue is the product of price p_i, the number of class i customers that are willing to buy at this price, $\Lambda_i \overline{F}_i\,(p_i)$, and the fillrate associated with this product, r_i. Equation (2.14) enforces the constraint that available capacity does not exceed sales, where the volume sold to class i customers is the product of class i demand and the fillrate corresponding to the product they purchase. The presence of the quality attribute r in the objective (2.13) and the capacity constraints (2.14) distinguishes this problem from the general problem (2.3), (2.4), (2.5), and (2.6).

2.3.4 No Capacity Constraint: Versioning of Information Goods

Consider a monopolistic software firm that serves a market of heterogeneous customers. To differentiate customers, the firm creates several versions of the software, and sells better versions at higher prices. Higher priced versions may have more features, a better user interface, and faster speed. Customers do not necessarily desire the fastest version, or the version with the most features, rather they seek to buy the cheapest product that satisfies their product and computational requirements. In such a setting it might be realistic to model customer choice behavior using threshold preferences. We will assume that the software product being sold by the firm is characterized by a one-dimensional quality attribute. The resulting revenue maximization problem is (2.3), (2.5), and (2.6).

2.3.5 Costly Quality Differentiation

So far quality differentiation has been costless, but this need not be the case. One popular example is in the sale of mp3 music players, such as the iPods. In particular, customers may have threshold preferences with respect to the mp3 player's storage capacity and seek to purchase the cheapest mp3 player with capacity above their specific threshold. The storage capacity of a mp3 player is a measure of the number of songs it can store, and customers that desire to carry along a larger number of songs have higher thresholds. The seller seeks to differentiate customers by selling mp3 players with different storage capacities at different prices, but in this case note that the higher quality products are more costly to produce. We will denote the marginal cost of a product with quality q_j as $s(q_j)$, where $s(\cdot)$ is a strictly increasing function of its argument. Then, the seller's product design problem can be formulated as follows:

$$\max_{p,q,M} \sum_{j=1}^{M} \sum_{i=1}^{N} (p_j - s(q_j))\, \Lambda_i\, \overline{F}_i\,(p_j)\, 1_{\{\chi_i(p,q)=j\}} \tag{2.17}$$

$$\text{s.t.} \sum_{j=1}^{M} \sum_{i=1}^{N} \Lambda_i \overline{F}_i (p_j) 1_{\{\chi_i(p,q)=j\}} \leq C, \tag{2.18}$$

$$0 \leq p < \infty, \quad 0 \leq q < \infty, \tag{2.19}$$

$$1 \leq M < \infty, \quad M \text{ integer.} \tag{2.20}$$

Formulation (2.17), (2.18), (2.19), and (2.20) is the same as (2.3), (2.4), (2.5), and (2.6), except for the objective, which is modified to reflect that the profit upon selling a unit of product j changes from p_j to $p_j - s(q_j)$.

2.4 Analysis of General Model

2.4.1 Model Assumptions

Assumption A: 1. The hazard rates of the valuation distributions, $h_i(v) := f_i(v)/\overline{F}_i(v)$, are decreasing in desired quality levels, i.e., $h_i(v) < h_{i+1}(v)$, $\forall v \in [0, \overline{v}_{i+1}]$, $1 \leq i < N$. 2. $r_i(\lambda) = \lambda \bar{F}_i^{-1}(\lambda/\Lambda_i)$ is strictly concave in λ for $i = 1, \ldots, N$.

Assumption B: $h_i(v)$ are decreasing and bounded below for $i = 1, \ldots, N$.

Discussion of modeling assumptions: To facilitate exposition and analysis we have assumed that the set of threshold quality levels is discrete and as such there are a finite and discrete set of customer types; this can be viewed as a discretization of a potentially continuous distribution of quality threshold preferences. Assumption A (1) on the hazard rates is equivalent to assuming that $\eta_i(v) < \eta_{i+1}(v)$, $\forall v \in [0, \overline{v}_{i+1}]$, $1 \leq i < N$, where $\eta_i = v f_i(v)/\overline{F_i}(v)$ is the demand elasticity of customer class i. That is, customers that desire higher quality levels are more inelastic than those desiring lower quality levels and, therefore, are less likely to walk away as the price is increased. The assumptions that hazard rates are monotonic and that the per class type in terms of arrival rates is concave are not restrictive. For example, the uniform, exponential, pareto, half-logistic Rayleigh distributions satisfy these assumptions. Finally Assumption B can be replaced with either (a) $h_i(v)$ are non-decreasing for $i = 1, \ldots, N$ or (b) $h_i(v)$ are decreasing and $\partial(1/h_i(v))/\partial v < 1$ for $i = 1, \ldots, N$.

2.4.2 Structural Results

First, we show that without loss of generality the firm only needs to offer products with quality levels in the set $\{\theta_1, \theta_2, \ldots, \theta_N\}$ and, second, that products with distinct prices must have distinct quality levels that are increasing in prices, and vice versa.

Lemma 1. *The following hold:*
(a) It suffices to offer quality levels that lie in the set $\{\theta_1, \theta_2, \ldots, \theta_N\}$.
(b) For any two distinct products (p_i, q_i) *and* (p_j, q_j), $q_i > q_j \Leftrightarrow p_i > p_j$.

Lemma 1 leads to the following corollary.

Corollary 1. *Suppose the firm offers* $1 \leq k \leq N$ *distinct products at qualities* $\theta_{i_1}, \ldots, \theta_{i_k}$, $1 \leq i_1 < i_2 < \ldots < i_k \leq N$, *at prices* $p_{i_1}, p_{i_2}, \ldots, p_{i_k}$, *respectively. Then* $p_{i_1} > p_{i_2} > \ldots > p_{i_k}$ *and (a)* $p_{i_1} < \bar{v}_{i_1}$ *and (b)* $p_{i_k} > 0$.

Lemma 2. *Any* $k \leq N$ *products partition the* N *customer classes into contiguous sets, i.e., if class* $i-1$ *and* $i+1$ *customers buy product* j*, then so do class* i *customers.*

Our next result shows that it is always optimal to offer the highest quality product.

Lemma 3. *The highest quality* θ_1 *is always offered.*

Lemma 3 leads to the following corollaries about the firm's one-product solution and the optimal product menu when the maximum valuations of all customer classes are the same.

Corollary 2. *The firm's one-product problem can be formulated as follows:*

$$\max_{p_1} \left\{ \sum_{i=1}^{N} p_1 \, \Lambda_i \, \overline{F}_i(p_1) \,\middle|\, \sum_{i=1}^{N} \Lambda_i \, \overline{F}_i(p_1) \; \leq \; C \right\}. \tag{2.21}$$

Corollary 3. *If* $\bar{v}_i = \bar{v}$, $i = 1, \ldots, N$, *then all classes buy a product from the firm.*

Lemmas 1–3 lead to the following formulation of the firm's revenue maximization problem.

Proposition 1. *The firm's problem (2.3), (2.4), (2.5), and (2.6) can be formulated as follows:*

$$\max_{p} \; \sum_{i=1}^{N} p_i \, \Lambda_i \, \overline{F}_i \, (p_i) \tag{2.22}$$

$$\text{s.t.} \; \sum_{i=1}^{N} \Lambda_i \, \overline{F}_i \, (p_i) \leq C, \tag{2.23}$$

$$p_N \leq p_{N-1} \leq \cdots \leq p_1, \quad i = 1, 2, \cdots, N, \tag{2.24}$$

$$p_i \leq \bar{v}_i \quad i = 1, 2, \ldots, N, \tag{2.25}$$

where p_i *denotes the price of the product being offered at quality* θ_i.

Proposition 1 simplifies considerably the firm's product design problem. The firm no longer needs to optimize over M and q, the number of qualities to offer and the vector of qualities, respectively, making the formulation (2.22), (2.23), and (2.24) more amenable to direct analysis.

Lemma 4. *Suppose qualities θ_m and θ_n are offered in the optimal solution, with $m+1<n$. Then qualities $\theta_l,\ m+1 \le l \le n-1$, are also offered.*

For homogeneous valuations, Lemma 4 leads to the following corollary.

Corollary 4. *If $\bar{v}_1 = \bar{v}_2 = \cdots = \bar{v}_N$, then it is optimal to offer exactly N products.*

2.4.3 Computation

The optimal solution to the revenue maximization problem can be easily computed. Under our assumption that $F_i(.)$ is continuous, this problem involves maximizing a continuous function over a compact set, and hence by Weierstrass theorem, an optimal solution exists. Compactness, note that the feasible set is bounded and that since the $F_i(.)$ are continuous, it is closed as well.

Instead of proceeding with a direct analysis of (2.22), (2.23), and (2.24), we will first restate the problem in terms of the demand rate vector as the optimization variable; this is typical in the revenue management literature. Specifically, for each class i, define $\lambda_i = \Lambda_i \overline{F}_i(p_i)$, so that $p_i = \overline{F}_i^{-1}(\lambda_i/\Lambda_i)$ because we assumed that $F_i(.)$ is continuous and increasing. We will also drop the monotonicity constraint (2.24) but later on verify that it is automatically satisfied by the optimal solution. The product design problem (2.22), (2.23), (2.24), and (2.25) reduces to

$$\max_{\lambda} \ \sum_{i=1}^{N} \lambda_i \, \overline{F}_i^{-1}\left(\frac{\lambda_i}{\Lambda_i}\right) \tag{2.26}$$

$$\text{s.t.} \ \sum_{i=1}^{N} \lambda_i \ \le C, \tag{2.27}$$

$$0 \le \lambda_i \le \Lambda_i, \quad i = 1, \ldots, N, \tag{2.28}$$

which is a concave maximization problem over a polyhedron and the same problem that arises in the context of multi-product pricing problem studied in Maglaras and Meissner (1998). The first-order conditions are both necessary and sufficient to characterize the optimal solution for (2.22), (2.23), and (2.24) or equivalently (2.26) and (2.27).

Proposition 2. *The optimal solution to the product design problem (2.22), (2.23), (2.24), and (2.25) is given by*

$$p_i = \frac{\overline{F}_i(p_i)}{f_i(p_i)} + \mu - \frac{\eta_i}{f_i(p_i)}, \tag{2.29}$$

$$\mu\left(C - \sum_{i=1}^{N} \Lambda_i \, \overline{F}_i(p_i)\right) = 0, \tag{2.30}$$

$$\mu \ge 0, \quad C - \sum_{i=1}^{N} \Lambda_i \, \overline{F}_i(p_i) \ge 0, \tag{2.31}$$

$$\eta_i(\bar{v}_i - p_i) = 0, \quad \eta_i \geq 0, \quad \bar{v}_i - p_i \geq 0. \tag{2.32}$$

Here μ is the Lagrange multiplier associated with the capacity constraint (2.23) and η_i is the Lagrange multiplier associated with the constraint $p_i \leq \bar{v}_i$. Following the assumptions made earlier in this section, the optimal prices satisfy the monotonicity constraint (2.24).

It is worth noting that the product design problem with fully rational customers making decisions according to a decision rule of the form of (2.1) is intractable with more than two customer types $(N > 2)$. This arises for two reasons. First, the product design problem cannot be reformulated as a function of the demand rates λ_i, and as a result the objective need not be concave in prices. Second, the quality decisions complicate the seller's problem substantially. One popular approach is to formulate the problem as a direct mechanism that captures the behavior embodied in (2.1) through the incorporation of appropriate incentive compatibility and individual rationality constraints, but these may not be convex in general; when $N = 2$ the problem simplifies using algebraic manipulations that cannot be exploited when $N > 2$.

2.4.4 k < N Products

For practical purposes the seller may only wish to restrict the number of products offered to the market. Such a strategy might be attractive when some customer types are similar or when administrative costs (not considered in our model) are high. It may also be driven by branding considerations (e.g., in the rationing example, the firm may not want to offer more than two products, so that customers that are rationed out do not discover that the product is available in a later period). We will assume that the firm seeks to offer $k < N$ products at qualities $\theta_{m_1}, \theta_{m_2}, \cdots, \theta_{m_k}$, with $1 \leq m_1 < m_2 < \cdots < m_k \leq N$, $m_{k+1} := N + 1$. Then, in a manner similar to Lemmas 1–3, it can be shown that it is optimal to set $m_1 = 1$, and $p_1 > p_2 > \cdots > p_k$. The firm's product design problem can be formulated as follows:

$$\max_{p} \sum_{l=1}^{k} p_l \sum_{j=m_l}^{m_{l+1}-1} \Lambda_j \bar{F}_j(p_l) \tag{2.33}$$

$$\text{s.t.} \sum_{l=1}^{k} \sum_{j=m_l}^{m_{l+1}-1} \Lambda_i \bar{F}_i(p_l)\, c \leq C, \tag{2.34}$$

$$0 \leq p_k \leq p_{k-1} \leq \ldots \leq p_1, \tag{2.35}$$

$$p_j \leq \bar{v}_{m_{j+1}-1}, \quad j = 1, \ldots, k. \tag{2.36}$$

In the following, we will assume that $\widetilde{h}_l(\cdot)$ satisfies Assumptions A and B, where

$$\widetilde{h}_l(v) = \frac{\sum_{j=m_l}^{m_{l+1}-1} f_j(v)\, \Lambda_j}{\sum_{j=m_l}^{m_{l+1}-1} \bar{F}_j(v)\, \Lambda_j}.$$

An example of a distribution that satisfies the above constraints is the exponential distribution with parameters $\alpha_1 < \alpha_2 < \cdots < \alpha_N$. Next, formulating the problem in terms of arrival rates, we obtain a concave maximization problem on a convex set, leading to the following characterization of optimal prices.

Proposition 3. *The optimal prices are characterized by*

$$p_l = \frac{\sum_{j=m_l}^{m_{l+1}-1} \overline{F}_j(p_l)\,\Lambda_j}{\sum_{j=m_l}^{m_{l+1}-1} f_j(p_l)\,\Lambda_j} + \mu - \frac{\eta_l}{\sum_{j=m_l}^{m_{l+1}-1} f_j(p_l)\,\Lambda_j}, \tag{2.37}$$

$$\mu\left(C - \sum_{l=1}^{k} \sum_{j=m_l}^{m_{l+1}-1} \overline{F}_j(p_l)\,\Lambda_j\right) = 0, \tag{2.38}$$

$$\mu \geq 0, \quad C - \sum_{l=1}^{k} \sum_{j=m_l}^{m_{l+1}-1} \overline{F}_j(p_l)\,\Lambda_j \geq 0, \tag{2.39}$$

$$\eta_l\,(v_{i_l-1} - p_l) = 0, \quad \eta_l \geq 0, \quad v_{i_l-1} - p_l \geq 0. \tag{2.40}$$

Here μ is the Lagrange multiplier associated with capacity constraint (2.34) and η_l is the Lagrange multiplier associated with constraint (2.36). The monotonicity of prices in (2.35) is ensured by our assumptions on $\widetilde{h}(\cdot)$.

The pricing problem given a preselected set of quality levels is simple, but the problem of identifying the optimal set of quality levels is combinatorial in nature. Since $m_1 = 1$ following Lemma 3, identifying the optimal k product solution requires solving $\binom{N-1}{k-1}$ problems. This can be computationally expensive for large k; however, solving the $k = 2$ problem requires solving $N-1$ problems to identify m_2 and is hence easily done.

2.5 Extensions

We next discuss a few extensions to the model studied in the previous section. First, we look at the example of capacity differentiation to illustrate how the baseline model can be extended to address the applications mentioned in Section 2.2. Second, we briefly review how one could treat a model with two or more quality attributes for which customers have dichotomous preferences. Finally, we offer some results on optimal product menu design in a duopoly setting.

2.5.1 Capacity Differentiation

The extension of the results of the previous section to the case where the quality attribute is the capacity that is allocated to each type of customer is fairly straightforward. For simplicity, in addition to the assumptions set forth in the previous section we will also restrict attention to valuation distributions for each customer type that have infinite support. In this setting, it is easy to verify that Lemmas 1 and 2 as well as their associated corollaries continue to hold. For Lemma 3 and 4, we need to slightly modify the proofs.

Lemma 5. *It is always optimal to offer the highest capacity product.*

The above result is slightly different from Lemma 3. In particular, we can no longer say that the optimal single product offering involves selling the highest capacity product. However, adding the highest capacity product to the existing product offering certainly increases revenues. Hence in the optimal product menu unconstrained by the number of products that are offered, the highest capacity product will always be offered.

Lemma 6. *Suppose the firm offers products at capacities θ_m and θ_n, where $m+1 < n$. Then it is optimal for the firm to offer products at capacity θ_l, $m+1 \leq l \leq n$.*

Together Lemmas 5 and 6 imply that if θ_k is the lowest capacity that is offered by the firm, then it is optimal to offer products with capacities $\theta_1, \ldots, \theta_{k-1}$. The following corollary shows that in fact it is optimal to offer all N products at capacities $\theta_1, \ldots, \theta_N$.

Corollary 5. *It is optimal to offer N products.*

Following Corollary 5, the service provider's revenue maximization problem can be reformulated as follows:

$$\max_{p} \sum_{i=1}^{N} p_i \, \Lambda_i \, \overline{F}_i \, (p_i) \tag{2.41}$$

$$\text{s.t.} \sum_{i=1}^{N} \Lambda_i \, \overline{F}_i \, (p_i) \, \theta_i \leq C, \tag{2.42}$$

$$0 \leq p_N < p_{N-1} < \cdots < p_1 < \infty. \tag{2.43}$$

We can solve the firm's revenue maximization problem (2.41), (2.42), and (2.43) by reformulating it in terms of arrival rates, wherein we obtain a concave maximization problem over a polyhedron. The first-order conditions lead to the following characterization of the optimal prices.

Lemma 7. *The optimal prices are given by $p_i = \overline{F}_i(p_i)/f_i(p_i) + \mu\theta_i$, $i = 1, \ldots, N$, where μ is the Lagrange multiplier associated with the capacity constraint.*

The $k < N$ products problem can also be solved in a similar fashion, though solving it now requires $\binom{N}{k}$ effort.

2.5.2 Multiple Quality Attributes

Our results extend naturally to the case where customers have threshold preferences with respect to more than one quality attribute. For simplicity, we discuss the two-attribute case here, which we will denote by θ_i and α_j, $i = 1, 2, \cdots, N_1$,

$j = 1, 2, \cdots, N_2$. Without loss of generality, we assume that $\infty > \theta_1 > \theta_2 > \cdots > \theta_{N_1} > 0$, $\infty > \alpha_1 > \alpha_2 > \cdots > \alpha_{N_2} > 0$, with higher values again denoting a desire for higher qualities. A type (i, j) customer is associated with the quality thresholds θ_i and α_j. Suppose the firm offers M products, where product l has price p_l and quality attributes q_l^1 and q_l^2. Then, a satisficing type (i, j) customer selects the following product:

$$\chi_{i,j}(p, q^1, q^2) = \begin{cases} \min_l p_l, & q_l^1 \geq \theta_i,\ q_l^2 \geq \alpha_j, \\ 0, & \text{otherwise.} \end{cases} \tag{2.44}$$

Analogous to the assumptions of Section 2.3, we assume that the supports of the valuation distributions satisfy the following ordering conditions $v_{1,j} > v_{2,j} > \cdots > v_{N,j}, \forall j$ and $v_{i,1} > v_{i,2} > \cdots > v_{i,M}, \forall i$. Also assume that

$$\frac{f_{i,1}(v)}{\overline{F}_{i,1}(v)} < \frac{f_{i,2}(v)}{\overline{F}_{i,2}(v)} < \ldots < \frac{f_{i,M}(v)}{\overline{F}_{i,M}(v)}, \quad \forall i,$$
$$\frac{f_{1,j}(v)}{\overline{F}_{1,j}(v)} < \frac{f_{2,j}(v)}{\overline{F}_{2,j}(v)} < \ldots < \frac{f_{N,j}(v)}{\overline{F}_{N,j}(v)}, \quad \forall j.$$

Hazard rates $f_{i,j}(p)/\overline{F}_{i,j}(p)$ are monotonic and $\lambda_{i,j}\overline{F}_{i,j}(\lambda_{i,j}/\Lambda_{i,j})$ is concave. Then, the results in Lemmas 1–4, Proposition 2, and their associated corollaries can be extended in a straightforward manner. As in Proposition 3, the k product problem can also be addressed, though the problem complexity increases significantly now (there are $\binom{N_1 N_2}{k}$ ways to choose k product quality combinations).

2.5.3 Duopoly

We finally consider some partial analysis of the case of two firms competing in a market with satisficing customers that satisfy the assumptions in Section 2.3. We examine the cases of simultaneous and sequential entry in order. As in Moorthy (1988), Shaked and Sutton (1982), and Wauthy (1996), we restrict attention to the case where each firm can offer only a single product and study a two-stage non-cooperative game. In the first stage, firms choose the quality level at which they seek to offer a product. In the second stage, given their and the competitor's quality, firms choose the prices at which to sell their product at. As in the above-mentioned papers, we focus on perfect Nash equilibria.

We begin by analyzing the second stage of the game, the price equilibrium. The two firms will not offer the same quality, else it will lead to a Bertrand game wherein profits would be zero. Hence we assume that firm 1 offers quality θ_i and firm 2 offers quality θ_j, $i < j$. Following Lemma 1 (which continues to hold), $p_i > p_j$ for two products to be offered. Then, the optimization problem for the firm offering quality θ_i can be written as follows:

$$\max_{p_i} \left\{ p_i \sum_{l=i}^{j-1} \overline{F}_l(p_i)\, \Lambda_l \;\middle|\; \sum_{l=i}^{j-1} \overline{F}_l(p_i)\, \Lambda_l \leq C,\ p_i \geq 0 \right\}. \tag{2.45}$$

Define p_1^* to be the optimal price in (2.45). The optimization problem for firm offering quality θ_j can be written as follows:

$$\max_{p_j}\left\{p_j \sum_{l=j}^{N} \overline{F}_l(p_j)\,\Lambda_l \;\middle|\; \sum_{l=j}^{N} \overline{F}_l(p_j)\,\Lambda_l \le C,\ p_1^* > p_j \ge 0\right\}. \tag{2.46}$$

Define p_2^* to be the optimal price in (2.46). The following lemma characterizes the Nash equilibrium in prices.

Lemma 8. *Equations (2.45) and (2.46) define a Nash equilibrium in prices (given fixed qualities).*

For simultaneous entry case we obtain the following result.

Proposition 4. *The unique product equilibrium occurs with firm 1 selecting quality θ_1 and firm 2 selecting quality θ_2, if the following condition is satisfied:*

$$\begin{aligned}&\max_{p_1}\{p_1\,\overline{F}_1(p_1)\,\Lambda_1 \mid \overline{F}_1(p_1)\,\Lambda_1 \le C\}\\ &\qquad \ge \max_{p_3}\left\{p_3 \sum_{l=3}^{N} \overline{F}_l(p_3)\,\Lambda_l \;\middle|\; \sum_{l=3}^{N} \overline{F}_l(p_3)\,\Lambda_l \le C, p_2^* > p_3 \ge 0\right\},\end{aligned} \tag{2.47}$$

where $p_2^* = \arg\max_{p_2}\{p_2\,\overline{F}_2(p_2)\,\Lambda_2 \mid \overline{F}_2(p_2)\,\Lambda_2 \le C\}$.

The analysis of the sequential entry is facilitated through the following notation:

$$R^1(l,p) = \{p\,\overline{F}_l(p)\,\Lambda_l \mid \overline{F}_l(p)\,\Lambda_l \le C\}, \tag{2.48}$$

$$R^1(l) = \max_p R^1(l,p), \quad p_1^l = \arg\max_p R^1(l,p), \tag{2.49}$$

$$\overline{R}^1(l,p) = \left\{\sum_{u=l+1}^{N} p\,\overline{F}_u(p)\,\Lambda_u \;\middle|\; \sum_{u=l+1}^{N} \overline{F}_u(p)\,\Lambda_u \le C, \quad p < p_1^l\right\}, \tag{2.50}$$

$$\overline{R}^1(l) = \max_{p<p_1^l} \overline{R}^1(l,p), \quad \overline{p}_1^l = \arg\max_{p<p_1^l} \overline{R}^1(l,p), \tag{2.51}$$

$$R^2(l,p) = \left\{\sum_{u=l}^{N} p\,\overline{F}_u(p)\,\Lambda_u \;\middle|\; \sum_{u=l}^{N} \overline{F}_u(p)\,\Lambda_u \le C, \quad p < \overline{p}_2^l\right\}, \tag{2.52}$$

$$R^2(l) = \max_{p<\overline{p}_2^l} R^2(l,p), \quad p_2^l = \arg\max_{p<\overline{p}_2^l} R^2(l,p), \tag{2.53}$$

$$\overline{R}^2(l,p) = \left\{\sum_{u=1}^{l-1} p\,\overline{F}_u(p)\,\Lambda_u \;\middle|\; \sum_{u=1}^{l-1} \overline{F}_u(p)\,\Lambda_u \le C\right\}, \tag{2.54}$$

$$\overline{R}^2(l) = \max_p \overline{R}^2(l,p), \quad \overline{p}_2^l = \arg\max_p \overline{R}^2(l,p). \tag{2.55}$$

$R^1(l,p)$ denotes the revenue achieved by firm 1, if it offers quality θ_l at price p and firm 2 decides to offer quality θ_{l+1}. $R^1(l)$ is the optimal revenue achieved in this case, and p_1^l denotes the revenue-maximizing price. $\overline{R}^1(l,p)$ denotes the revenue achieved by firm 2, if firm 1 offers quality θ_l at price p_1^l, and firm 2 offers quality θ_{l+1} at price p. $\overline{R}^1(l)$ denotes the optimal revenue achieved in this case, and $\overline{p}_1^l$ denotes the corresponding revenue-maximizing price. $R^2(l,p)$ denotes the revenue achieved by firm 1, if it offers quality l at price $p < \overline{p}_2^l$ and firm 2 decides to offer quality θ_1 at price $\overline{p}_2^l$. $R^2(l)$ is the optimal revenue achieved in this case, and p_2^l denotes the revenue-maximizing price. $\overline{R}^2(l,p)$ denotes the revenue achieved by firm 2, if firm 1 offers quality θ_l, and firm 2 offers quality θ_1 at price p. $\overline{R}^2(l)$ denotes the optimal revenue achieved in this case, and $\overline{p}_2^l$ denotes the corresponding revenue-maximizing price. The following proposition characterizes the optimal qualities to offer.

Proposition 5. *The first entrant chooses to offer quality*

$$i = \arg\max_{l=1,2,\ldots,N} R(l), \tag{2.56}$$

$$R(l) = \begin{cases} R^1(l), & \textit{if } \overline{R}^1 \geq \overline{R}^2, \\ R^2(l) & \textit{otherwise}. \end{cases} \tag{2.57}$$

The quality chosen by the second entrant then is θ_1 *if* $\overline{R}_i^1 < \overline{R}_i^2$, *and* θ_{i+1} *otherwise.*

We note that while in the simultaneous case the two best quality products are offered if an equilibrium exists, in the sequential entry case, neither of the two best qualities may be offered. This is in contrast with the optimal two-product solution of a monopolist firm, where the first product is always offered at the best quality, while the quality of the second product depends upon the problem parameters.

2.6 Concluding Remarks: Satisficers vs. Utility Maximizers

In this chapter, we have analyzed the product design problem for a seller facing a market of satisficing customers. The product design problem is tractable and enjoys several nice structural properties about the optimal number of products, the quality levels of the offered products, the structure of the product manu if the seller wants to restrict the number of offered products, and the structure of the optimal policy in a simplified duopoly setting. We also note that the ability to solve for the optimal menu in a multi-type market is a significant improvement over what can be done with classical models of vertical and horizontal product differentiations or mechanism design approaches for utility maximizing customers.

Satisficing provides a plausible approach to model bounded rationality in some revenue management and operations management contexts that we believe has both analytical and practical importance. An obvious issue that we have not addressed concerns the empirical validation of the satisficing customer choice behavior, which is an interesting problem that has been only partially addressed in the marketing and psychology literature.

2.7 Proofs

Proof of Lemma 1: Part (a). Since any customer class i is indifferent between the quality levels that lie in the interval $(\theta_{l-1}, \theta_l]$, $l = 1, \ldots, N$, where $\theta_0 := \overline{\theta}$, at most one price can be charged for any quality level in $(\theta_{l-1}, \theta_l]$, $l = 1, \ldots, N$. Hence, offering one quality level in $(\theta_{l-1}, \theta_l]$, $l = 1, \ldots, N$, suffices, which without loss of generality, we can fix to θ_l. Part (b). Following (a), the quality levels q_i and q_j lie in the set $\{\theta_1, \theta_2, \ldots, \theta_N\}$. Suppose $q_i > q_j$ while $p_i < p_j$. Then, every customer strictly prefers product i over product j. Hence the firm can drop product j from its product line without affecting its revenues. This would contradict our assumption that the firm only offers products that generate non-zero demand, and so $p_i > p_j$. Suppose now that $p_i > p_j$ but $q_i < q_j$. In this case, all customers strictly prefer product j to product i, which therefore generates zero demand. Again, this contradicts our assumption that the firm only offers products that generate non-zero demand and hence $q_i > q_j$. □

Proof of Corollary 1: The monotonicity of prices, $p_{i_1} > p_{i_2} > \cdots > p_{i_k}$, follows from Lemma 1. Part (a). If $p_{i_1} \geq \overline{v}_{i_1}$, then $\overline{F}_{i_1}(p_{i_1}) = 0$, implying that nobody will purchase this product, and it can be dropped. This violates our assumption that only products that offer a non-zero demand are offered. Hence, $p_{i_1} < \overline{v}_{i_1}$. Part (b). Suppose $p_{i_k} = 0$. Consider setting p_{i_k} to $0.5 \min\{p_{i_{k-1}}, \overline{v}_{i_k}\} > 0$ wherein the aggregate demand decreases while revenues increase. Note that $p_{i_{k-1}} > 0$, since products k and $k-1$ are distinct. Hence $p_{i_k} > 0$ in the optimal solution. □

Proof of Lemma 2: Since type $i-1$ buys product j, $q_j \geq \theta_{i-1} > \theta_i$, i.e., the quality of product j is higher than the quality threshold for type i. Since type $i+1$ buys product j, $p_j = \min_{q_l \geq \theta_{i+1}} p_l \leq \min_{q_l \geq \theta_i} p_l$, i.e., product j is the cheapest product offered by the firm with quality greater than or equal to θ_i. Hence it is optimal for type i to buy product j. □

Proof of Lemma 3: Let θ_k, $k > 1$, be the highest quality offered at price p to the market in the optimal solution. Also suppose that customers from classes l, $k \leq l \leq i$ are currently buying this product. Consider increasing the quality of the offered highest quality product from θ_k to θ_1 and increasing its price from p to $p + \varepsilon$, $\varepsilon > 0$, such that $\sum_{l=1}^{i} \overline{F}_l(p+\varepsilon)\Lambda_l = \sum_{l=k}^{i} \overline{F}_l(p)\Lambda_l$. The left hand side is continuous and decreasing in ε, exceeds the right hand side for $\varepsilon = 0$, and is less than the right hand side for $\varepsilon = \infty$. Hence, such an $\varepsilon > 0$ exists. Since the demand does not change while revenues increase (we increased the price), the original solution cannot be optimal and we have a contradiction. □

Proof of Corollary 2: Following Lemma 3, if a single product is offered by the firm, then it is offered at the highest quality θ_1. The one-product problem formulation then follows. □

Proof of Corollary 3: Since at least one product is offered, following Lemma 3, the highest quality product is offered. Let p_1 denote its price. Then $p_1 < \overline{v}$, else this product will generate zero demand. Since any other products would be offered at a lower quality level, and hence price (following Lemma 1), at least some customers from each class would buy from the firm. □

Proof of Proposition 1: Following Lemma 3, quality θ_1 is always offered. Hence, all customer classes $1, \cdots, N$ would buy a product from the firm, subject to their valuations exceeding the price p_1. If $k < N$ products are offered in the optimal solution at qualities $\theta_{m_1}, \theta_{m_2}, \cdots, \theta_{m_k}$, with $m_1 < m_2 < \cdots < m_k \leq N$, $m_1 = 1, m_{k+1} := N+1$, and prices $p_{m_1} > p_{m_2} > \cdots > p_{m_k}$, then setting prices to be $p_j = p_{m_i}, m_i \leq j < m_{i+1}$, $i = 1, \cdots, k$, in the above formulation would lead to the same solution. Finally, any solution of (2.22), (2.23), and (2.24) is consistent with customer behavior, in that type i customers would buy the product priced at p_i. Hence the formulation is correct. □

Proof of Lemma 4: Suppose it is optimal for the firm to offer $k < N$ products (lemma holds trivially if $k = N$). Then there exist indices $1 \leq i_1 < i_2 < \cdots < i_k \leq N$ such that product l, $1 \leq l \leq k$, is being offered at quality θ_{i_l}. Following Lemma 3, $i_1 = 1$. Suppose there exist indices m, n such that $m+1 < n$, $i_l = m$, $i_{l+1} = n$ for some $1 \leq l \leq k-1$. These correspond to products with qualities θ_m and θ_n, respectively. In case such indices do not exist (since $k < N$ and $i_1 = 1$, this case occurs only when the k products are offered at qualities $\theta_1, \ldots \theta_k$), the lemma holds. Even then for the first case of the following two, we set $m = k$, $n = N+1$, $p_{N+1} = 0$, $\theta_{N+1} = 0$. For the second case, we consider only the possibility where such indices do exist. Let us denote the prices of these two products by p_m and p_n, respectively, with $p_m > p_n$ (since $\theta_m > \theta_n$ and following Lemma 1). There are two cases to consider.

Case (a): $p_m < \bar{v}_{m+1}$: Consider adding a product at quality level θ_{m+1} and price $p_m - \delta$, $\delta > 0$ such that $p_m - \delta > p_n$ and increasing the price of the product being offered at quality θ_m from p_m to $p_m + \varepsilon$, $\varepsilon > 0$, such that $p_m + \varepsilon < \bar{v}_m$ and $p_m + \varepsilon < p_{i_{l-1}}$, where $p_{i_{l-1}}$ is the price of the θ_{l-1} best quality product offered by the firm, if any, and ∞ otherwise. The change in demand $\Delta D = \Lambda_m \overline{F}_m(p_m + \varepsilon) + \sum_{u=m+1}^{n-1} \Lambda_u \overline{F}_u(p_m - \delta) - \Lambda_m \overline{F}_m(p_m) - \sum_{u=m+1}^{n-1} \Lambda_u \overline{F}_u(p_m)$. Using the first-order Taylor expansion, we can write $\Delta D = -\varepsilon \Lambda_m f_m(p_m) + \delta \sum_{u=m+1}^{n-1} \Lambda_u f_u(p_m) + o(\varepsilon) + o(\delta)$. Similarly, the change in revenue $\Delta R = \Lambda_m (p_m + \varepsilon) \overline{F}_m(p_m + \varepsilon) + (p_m - \delta) \times \sum_{u=m+1}^{n-1} \Lambda_u \overline{F}_u(p_m - \delta) - \Lambda_m p_m \overline{F}_m(p_m) - p_m \sum_{u=m+1}^{n-1} \Lambda_u \overline{F}_u(p_m)$. Again, $\Delta R = \varepsilon \Lambda_m (\overline{F}_m(p_m) - p_m f_m(p_m)) + \delta \sum_{u=m+1}^{n-1} \Lambda_u (p_m f_u(p_m) - \overline{F}_u(p_m)) + o(\varepsilon) + o(\delta)$.

We want to show that there exist δ, ε, small such that $\Delta D < 0$, $\Delta R > 0$. To this end, choose δ such that $\delta \sum_{u=m+1}^{n-1} \Lambda_u f_u(p_m) = \gamma \varepsilon \Lambda_m f_m(p_m)$, where $0 < \gamma < 1$. This implies that $\Delta D = -\varepsilon \Lambda_m f_m(p_m)(1-\gamma) + o(\varepsilon) = -\delta[(1-\gamma)/\gamma] \sum_{u=m+1}^{n-1} \Lambda_u f_u(p_m) + o(\delta)$, which is < 0 when ε (or equivalently δ) is small enough. Substituting this value of δ and simplifying, we get

$$\Delta R = \frac{\varepsilon \Lambda_m f_m(p_m) p_m}{\sum_{u=m+1}^{n-1} \Lambda_u f_u(p_m)} \left[\sum_{u=m+1}^{n-1} \Lambda_u f_u(p_m) \left(\frac{1}{\eta_m(p_m)} - 1 + \gamma - \frac{\gamma}{\eta_u(p_m)} \right) \right] + o(\varepsilon).$$

Now

$$\eta_m(p_m) < \eta_{m+1}(p_m) \le \eta_u(p_m) \;\Leftrightarrow\; \frac{1}{\eta_m(p_m)} > \frac{1}{\eta_{m+1}(p_m)} \ge \frac{1}{\eta_u(p_m)}.$$

Hence, for ε sufficiently small, it suffices to show that

$$\frac{1}{\eta_m(p_m)} - 1 > \gamma\left(\frac{1}{\eta_{m+1}(p_m)} - 1\right),$$

which holds from above and the fact that we can choose any γ that satisfies $0 < \gamma < 1$.

Case (b): $p_m > \bar{v}_{m+1}$: In this case, classes $m+1 \le u < n$ do not buy any product. Consider adding a product at quality level θ_{m+1} and price $\bar{v}_{m+1} - \varepsilon$, $\varepsilon > 0$, and increasing the price of the product offered at θ_n from p_n to $p_n + \delta$, $\delta > 0$ such that $p_n + \delta < \bar{v}_n$ and $\bar{v}_{m+1} - \varepsilon > p_n + \delta$. Let θ_r be the next best quality after θ_n that is offered by the firm (set it to $r = N+1$, $\theta_{N+1} = 0$, $p_{N+1} = 0$, as mentioned earlier, if there is none). The change in demand $\Delta D = \sum_{u=m+1}^{n-1} \overline{F}_u(\bar{v}_{m+1} - \varepsilon)\Lambda_u + \sum_{u=n}^{r-1} \overline{F}_u(p_n + \delta)\Lambda_u t_j - \sum_{u=n}^{r-1} \overline{F}_u(p_n)\Lambda_u$. Using the first-order Taylor expansion, $\Delta D = \varepsilon \sum_{u=m+1}^{n-1} f_u(\bar{v}_{m+1})\Lambda_u - \delta \sum_{u=n}^{r-1} f_u(p_n)\Lambda_u + o(\varepsilon) + o(\delta)$. Similarly, the change in revenue, $\Delta R = \sum_{u=m+1}^{n-1} \overline{F}_u(\bar{v}_{m+1} - \varepsilon)\Lambda_u(\bar{v}_{m+1} - \varepsilon) + \sum_{u=n}^{r-1} \overline{F}_u(p_n + \delta)\Lambda_u(p_n + \delta) - \sum_{u=n}^{r-1} \overline{F}_u(p_n)\Lambda_u p_n$, which can be written as $\Delta R = \bar{v}_{m+1}\varepsilon \sum_{u=m+1}^{n-1} f_u(\bar{v}_{m+1})\Lambda_u - \delta \sum_{u=n}^{r-1} \Lambda_u(p_n f_u(p_n) - \overline{F}_u(p_n)) + o(\varepsilon) + o(\delta)$. Choose $\varepsilon \sum_{u=m+1}^{n-1} f_u(\bar{v}_{m+1})\Lambda_u = \gamma\delta \sum_{u=n}^{r-1} f_u(p_n)\Lambda_u$, with $0 < \gamma < 1$, so that

$$\Delta R = \delta \sum_{u=n}^{r-1} f_u(p_n)\Lambda_u p_n \left(\frac{\bar{v}_{m+1}\gamma}{p_n} - 1 + \frac{\overline{F}_u(p_n)}{p_n f_u(p_n)}\right) + o(\delta).$$

For δ sufficiently small, a sufficient condition for $\Delta R > 0$ is that $\bar{v}_{m+1}\gamma/p_n > 1$, which is true if we choose $\gamma > p_n/\bar{v}_{m+1}$. This is possible, since the only restriction on our choice of γ was $0 < \gamma < 1$, and $p_n < \bar{v}_n \le \bar{v}_{m+1}$.

Hence in both cases, we obtain a contradiction. □

Proof of Corollary 4: In the proof of Lemma 4, under the assumption that $\bar{v}_i$ = constant for all $i = 1, \ldots, N$, the second case in the proof never arises. The proof of the first part is applicable for all k, $1 \le k < N$, irrespective of whether there are holes in the product offering or not. Hence we know that $\forall k < N$, offering $k+1$ products over k products increases revenues. Also from Lemma 1, we know that it suffices to offer at most N products. Hence, it is optimal to offer exactly N products. □

Proof of Lemma 5: Suppose that the highest quality product is being offered at capacity θ_k and price p_k in the optimal solution, where $k > 1$ (else the lemma holds). Note that $p_k < \infty$. Also suppose that the next highest quality product was being offered at θ_m, $m > k$. (Set $m = N+1$ if no other product is offered.) Consider introducing an additional product at capacity θ_1 and price $p_1 = p_k + \varepsilon$, $\varepsilon > 0$, such that $p_1 > p_k(\theta_1/\theta_k)$. Also, increase the price of product with capacity θ_k to $p_k + \delta$, $\delta > 0$, such that $\Delta D = \theta_1 \sum_{l=1}^{k-1} \Lambda_l \overline{F}_l(p_k + \varepsilon) + \theta_k \sum_{l=k}^{m-1} \Lambda_l [\overline{F}_l(p_k + \delta) - \overline{F}_l(p_k)] = 0$.

The first term is positive and decreases as ε increases, while the second term is negative and decreases as δ increases. Hence, there exist $\varepsilon > 0$, $\delta > 0$, such that $p_k + \delta < \bar{v}_k$, $p_k + \varepsilon < \bar{v}_1$, and $\Delta D = 0$. As a result, demand is unchanged, while the cost per unit capacity for products sold to classes $1 \ldots k$ increases to $\min\{p_1/\theta_1, (p_k+\delta)/\theta_k\} > p_k/\theta_k$. Hence the total revenue increases via the introduction of this product at θ_1. □

Proof of Lemma 6: As in Lemma 4, consider two indices m, n where $m+1 < n$, such that products are offered at θ_m and θ_n, but none in between. Following Corollary 1, $p_m < \infty$. Consider adding a product at θ_{m+1} and price $p_m - \delta$ while increasing the price of the product with capacity θ_m to $p_m + \varepsilon$. Then,

$$\begin{aligned}\Delta D = &\sum_{l=m+1}^{n-1} \Lambda_l \overline{F}_l(p_m - \delta)\theta_{m+1} + \Lambda_m \overline{F}_m(p_m + \varepsilon)\theta_m \\ &- \sum_{l=m}^{n-1} \Lambda_l \overline{F}_l(p_m)\theta_m = -\varepsilon\theta_m\Lambda_m f_m(p_m) \\ &+ \delta\theta_{m+1} \sum_{l=m+1}^{n-1} \Lambda_l f_l(p_m) + (\theta_{m+1} - \theta_m) \sum_{l=m+1}^{n-1} \Lambda_l \overline{F}_l(p_m) + o(\varepsilon) + o(\delta).\end{aligned}$$

Since $\theta_{m+1} < \theta_m$, choose δ sufficiently small so that $\Delta D < 0$. Similarly,

$$\begin{aligned}\Delta R = &\sum_{l=m+1}^{n-1} \Lambda_l \overline{F}_l(p_m - \delta)(p_m - \delta) + \Lambda_m \overline{F}_m(p_m + \varepsilon)(p_m + \varepsilon) \\ &- \sum_{l=m}^{n-1} \Lambda_l \overline{F}_l(p_m) p_m \\ = &\,\varepsilon\Lambda_m \overline{F}_m(p_m) - \varepsilon\Lambda_m p_m f_m(p_m) \\ &+ \sum_{l=m+1}^{n-1} \Lambda_l \delta[-\overline{F}_l(p_m) + p_m f_l(p_m)] + o(\varepsilon) + o(\delta).\end{aligned}$$

Hence $\Delta R > 0$ if $\varepsilon\Lambda_m \overline{F}_m(p_m)(1 - \eta_m(p_m)) > \delta \sum_{l=m+1}^{n-1} \Lambda_l \overline{F}_l(p_m)(1 - \eta_l(p_m))$.

Define $A = \Lambda_m \overline{F}_m(p_m)(1 - \eta_m(p_m))$ and $B = \sum_{l=m+1}^{n-1} \Lambda_l \overline{F}_l(p_m)(1 - \eta_l(p_m))$. From our assumption on elasticities $\eta_m < \eta_l, m \le l \le n$. There are three possibilities:

(i) $A > 0, B > 0$: choose δ sufficiently small (compared to ε) $\implies \Delta R > 0$.

(ii) $A > 0, B < 0$: $\implies \Delta R > 0$.

(iii) $A < 0, B < 0$: choose ε sufficiently small (compared to δ) $\implies \Delta R > 0$.

Hence introducing a product at θ_{m+1} increases revenues. □

Proof of Corollary 5: In the proof of Lemma 6, substituting $n = N+1$, and introducing a dummy product with $\theta_{N+1} = 0$, $p_{N+1} = 0$, does not affect line of argument. Hence we conclude that if only first k capacities are being offered, introducing a product at capacity θ_{k+1} also increases revenues. Applying this argument iteratively and following Lemma 1, we conclude that it is optimal to offer exactly N products. □

Proof of Lemma 8: Firm 1 has no incentive to change its price, since given the quality θ_i of its product, this is the optimal price for it to charge subject to its capacity. Firm 2 needs to offer a lower price than firm 1 to be able to generate non-zero revenues, since $\theta_i > \theta_j$. Hence, given its quality θ_j and capacity C, p_2^* is the optimal price for firm 2 to charge. Finally the resulting customer choice behavior is consistent with the formulation in (2.45) and (2.46). □

Proof of Proposition 4: Suppose the product equilibrium occurs at $1 < i < j \leq N$. Given choice of quality θ_j by firm 2, firm 1 will find it advantageous to offer quality θ_1, for it increases revenues when the price equilibrium with product qualities fixed is considered. Hence, in the Nash equilibrium, $i = 1$. Next consider the case where $j > 1$. In this case, given that firm 1 chooses to offer quality θ_1, firm 2 revenue would increase if it offers quality θ_2 instead of θ_j, given the price equilibrium that would occur with these qualities. Hence $j = 2$ in the Nash equilibrium. Next we consider whether $i = 1, j = 2$ constitutes a Nash equilibrium. Clearly, firm offering θ_2 does not have an incentive to deviate. As for the firm offering θ_1, the best alternative is to offer quality θ_3 instead. This happens only if

$$\max_{p_1}\{p_1\,\overline{F}_1(p_1)\,\Lambda_1 \mid \overline{F}_1(p_1)\,\Lambda_1 \leq C\}$$
$$< \max_{p_3}\left\{p_3 \sum_{l=3}^{N} \overline{F}_l(p_3)\,\Lambda_l \;\middle|\; \sum_{l=3}^{N} \overline{F}_l(p_3)\,\Lambda_l \leq C, p_2^* > p_3 \geq 0\right\},$$

where $p_2^* = \arg\max_{p_2}\{p_2\,\overline{F}_2(p_2)\,\Lambda_2 \mid \overline{F}_2(p_2)\,\Lambda_2 \leq C\}$. □

Proof of Proposition 5: Since firm 1 chooses its quality first, and with the knowledge that firm 2 will subsequently choose the optimal quality to offer following firm 1's choice, there are two situations to consider. Given firm 1's choice of quality θ_l, firm 2 would offer either a better quality, in which case it is optimal for firm 2 to offer θ_1, or a worse quality, in which case it is optimal for firm 2 to offer θ_{l+1}. The revenues resulting for firm 2 in these two situations are denoted by $\overline{R}^1(l)$ and $\overline{R}^2(l)$ for firm 2, respectively. Firm 2 chooses quality θ_1 if $\overline{R}^1(l) \geq \overline{R}^2(l)$, in which case the revenue achieved by firm 1 is given by $R^1(l)$ in equilibrium. If $\overline{R}^1(l) < \overline{R}^2(l)$, then firm 2 chooses quality θ_{l+1}, and consequently, firm 1 obtains $R^2(l)$ in revenue in equilibrium. This leads to (2.57). Given the optimal revenue achievable by firm 1 if it offers quality θ_1 to the market, firm 1 then optimizes over qualities θ_l, $l = 1,\ldots,N$, to identify the optimal quality to offer, as summarized by (2.56). □

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